

# A Kalman filter approach to dynamic OD flow estimation for urban road networks using multi-sensor data

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## SUMMARY

Considerable efforts have been devoted to the development of dynamic origin-destination (OD) estimation models, which are a key step to realizing self-adaptive traffic control systems for urban traffic management. However, most of the models proposed to date estimate OD flows based on a single traffic data source, and their performance is limited by the coverage and accuracy of traffic sensors. The inherent difficulty in estimating the dynamic traffic assignment matrix means that dynamic OD estimation remains a challenge for real-life applications. This paper proposes the use of a Kalman filter for dynamic OD estimation using multi-source sensor data. The dynamic characteristic of changing OD flow over time is analyzed, and the problem of dynamic OD estimation is converted to a problem of estimating OD structural deviation. The resulting dynamic relationship between traffic volume and OD structural deviation is then used to establish the Kalman filter model. An improved traffic assignment approach is developed and embedded into the measurement equation of the Kalman filter model to enable dynamic updating of the traffic assignment matrix. A dual self-adaptive mechanism based on the Kalman filter is used to calibrate the model. The proposed method was implemented on a real-life traffic network in the downtown area of Kunshan City, China. The results show that the proposed method is more accurate than, and outperforms, the traditional link-volume-based and turning-movement-based methods. Copyright © 2014 John Wiley & Sons, Ltd.

**KEY WORDS:** dynamic OD estimation; urban network; traffic sensors; traffic simulation; dynamic traffic assignment; Kalman Filter

## 1. INTRODUCTION

Self-adaptive traffic control, by effectively improving the performance of urban road traffic management, plays an essential role in intelligent transportation systems. The performance of a self-adaptive traffic control system relies heavily on accurate origin-destination (OD) demand data. Even though today's modern urban road networks incorporate traffic sensors, the coverage and accuracy of these traffic sensors are never enough to monitor each vehicle's movement and accurately estimate traffic demand in a timely manner. Because of the poor quality and insufficient data produced by existing traffic sensors, dynamic OD estimation remains a challenge for transportation engineers and planners.

Existing dynamic OD estimation models typically estimate vehicle OD flows based on previously collected traffic data and can be classified broadly into two categories: non-assignment-based and assignment-based approaches. The non-assignment-based approach estimates OD flows according to the relationship between inbound and outbound traffic volumes on the network based on the law of traffic volume conservation [1, 2]. However, this approach suffers from limitations when describing

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complex traffic conditions, because it does not consider microscopic traffic flow behaviors such as vehicle queuing and signal delays. Thus, the non-assignment-based methods are more suitable for estimating OD flows at an independent intersection or in a 'closed' network such as a simple freeway network, where all link volumes can be easily observed.

Of the two approaches, the assignment-based approach is more capable of handling OD estimation on a large-scale urban network. Even though traffic sensors may not be installed on each road link, the assignment-based approach can still perform effectively if the dynamic mapping relationship between observed traffic volumes and OD flows can be accurately established through a dynamic assignment matrix, which can be derived by analytical methods or traffic simulators. Regardless of the method used to obtain the assignment matrix, the assignment-based approach can be classified primarily into five major types of models, namely generalized least square models [3, 4], bi-level models [5–8], Bayesian theory-based models [9], maximum entropy models [10], and state-space models [11–14]. The first four of these typically utilize optimization methods to minimize the discrepancy between the observed and assigned traffic volumes under various constraints. Because the temporal relationship of OD flows is difficult to account for in real time, these model types are commonly used for offline OD estimation. The state-space model, however, has been widely used in real-time transportation engineering applications, such as travel time estimation [15], traffic volume prediction [16], and traffic safety [17]. In order to describe the time-varying characteristics of OD flows in such a way that the dynamic OD can be adaptively estimated, various state-space models have been developed. Compared with the first four model types, the state-space models minimize the discrepancy between observed and assigned traffic volumes in an incremental fashion that allows the estimated results to be updated as additional data becomes available.

In this context, Okutani [11] used an auto-regressive process to describe OD dynamics by first implementing a state-space model for dynamic OD estimation. The results showed that accurate estimation depends on sufficient traffic volume and historical OD data. Subsequent studies [12–14] mainly focused on two ways to improve the accuracy of state-space models: improving the state equation to better describe OD dynamics over time and improving the measurement equation to better describe the dynamic mapping relationship between observed traffic volumes and OD flows.

As the model by Okutani [11] is not capable of capturing OD dynamics from historical OD flows, Ashok and Ben-Akiva [12] attempted to improve the state equation by using an autoregressive process to describe OD dynamics that was based on the deviations between actual and historical OD flows. Ashok and Ben-Akiva [13] then went on to define a state vector in terms of deviations in the departure rates from each origin and the shares headed to each destination, and to design a new state equation to utilize the differential temporal variation of trips and shares. Their improved method performed well, although some loss of accuracy was found in the filtered estimates. An alternative approach to capture the possible structural OD deviations between actual and historical OD flows by Zhou and Mahmassani [14] used a polynomial trend filter to represent the dynamic OD changes, with the accuracy of the model depending on the number of polynomial terms. The major drawback of this method, however, is that the optimum number of polynomial terms is difficult to determine. Additionally, Zhou and Mahmassani's study assumed that the OD structural deviation is positively proportional to the length of the time interval, which may not be true in all situations because the changes in the OD flow are not always linear.

The second way to improve the accuracy of state-space model is to enhance the measurement equation to track more closely the relationship between OD flows and observed traffic volumes. By considering the stochasticity of dynamic OD estimation, Ashok and Ben-Akiva [18] established a dynamic mapping relationship between OD flows and link volumes in the measurement equation. Their model provides a systematic way to model the uncertainty of travel time and route-choice fractions. Similarly, Antoniou *et al.* [19] proposed an online calibration approach for the nonlinear state-space model. Their results demonstrated that the online calibration of model parameters can indeed improve estimation accuracy. However, as pointed out by Ashok and Ben-Akiva [18], the time-varying characteristic of the mapping relationship between traffic volumes and OD flows is difficult to estimate, though many approaches have been suggested.

Traffic data is now widely collected by multiple sources of traffic sensors, including microwave, loops, cameras [20], and cellular data [21]. The types of data collected by each type of sensor are

different: microwave detectors mainly detect the volume, speed, and occupancy of traffic; cameras or loops installed at intersections detect turning movement volumes; and mobile phone data can be used to detect vehicle positions and speed. However, regardless of the different data types collected, the raw data must be aggregated and converted to traffic flow parameters such as speed, volume, occupancy, and travel time if it is to be useful. The integration of multiple sources of data is a feature of a number of OD estimation applications [22–25]. For example, Alibabai and Mahmassani [23] presented a dynamic OD estimation model based on a bi-level optimization method that utilized both turning movement volumes and link volumes. These studies all demonstrated that multisource traffic data can be used effectively to improve the accuracy of dynamic OD estimation.

In this paper, a dynamic OD estimation method based on a state-space model (i.e., a Kalman filter model) is proposed. The study's contributions and innovations cover four main aspects: (1) multi-sensor data, that is, link volumes detected by microwave sensors and turning movement volumes detected by video cameras, are integrated into the dynamic OD estimation model framework; (2) unlike the study by Alibabai and Mahmassani [23], the proposed method considers the structural OD deviations between actual and regular OD flows in the state equation of the model, so the resulting measurement equation establishes a mapping relationship between OD flows and turning movement volumes that reflects changes in the OD flow due to changing traffic volumes; (3) an improved traffic assignment approach is developed to integrate the turning movement volumes into the online updating process of the assignment matrix in the measurement equation, in which the assignment matrix is calculated using real-time travel times to reveal the fluctuations in the route impedance; and (4) a dual self-adaptive update mechanism is developed for the proposed Kalman filter model in order to represent weekly changes in the historical OD flows on urban road networks.

## 2. METHODOLOGY

This section presents the proposed method based on the state-space model, that is, a Kalman filter model. First, the framework is described, and then the proposed Kalman filter model – including the state equation and measurement equation – is designed and solved. The details of the proposed method are as follows.

### 2.1. Framework

Typically, urban road traffic follows time-of-day and day-of-week patterns. Thus, it is reasonable to assume that OD flow follows similar traffic patterns. At the same time, OD flow may also vary with real-time traffic fluctuations. Therefore, by defining the structural deviation of OD flow as the difference between actual and regular pattern OD flows, the dynamic OD can be processed as the sum of regular OD, time-varying structural deviations, and random errors as

$$\text{Estimated OD flow} = \text{Regular OD} + \text{Structural deviation} + \text{Random error} \quad (1)$$

Because the regular time-of-day and day-of-week OD patterns can be obtained from historical traffic data, the challenge of dynamic OD estimation using a Kalman filter model is to design appropriate state and measurement equations for accurately estimating the structural deviation of OD flows. To achieve this, the state equation of the proposed Kalman filter model must first be derived based on Equation (1) for prior estimates of the structural deviation of OD flows. The relationships between OD flows and traffic volumes are then analyzed to establish the measurement equation of the Kalman filter model for posterior estimates of the structural deviation of OD flow. Unlike previous studies, which used either link or turning movement volumes, the proposed method integrates both the link and turning volumes into the measurement equation to build the traffic assignment proportion matrix, where the travel times are estimated from the detector data and used as the route impedances. Finally, a dual self-adaptive mechanism is designed to solve the Kalman filter model and to update the regular OD flows. The framework of the proposed method for recursively estimating the OD flows in real time is shown in Figure 1.

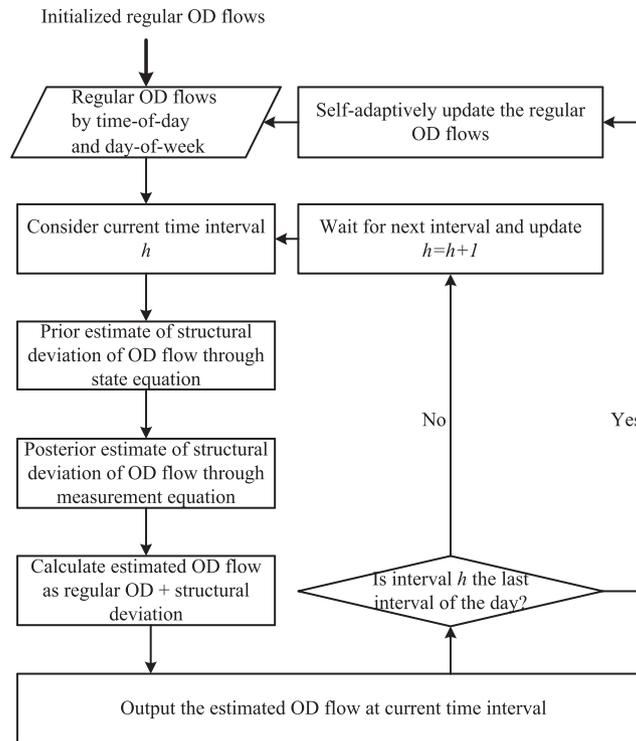


Figure 1. Framework of the proposed method.

2.2. Kalman filter design

2.2.1. Establishment of the state equation

Let  $T_h$  and  $T_h^H$  denote the dynamic OD and regular OD flows, respectively, for time interval  $h$ ,  $X_h$  denote the structural deviation of OD flow in time interval  $h$ , and  $\epsilon_h$  denote the random error in time interval  $h$ . Equation (1) can now be written as

$$T_h = T_h^H + X_h + \epsilon_h \tag{2}$$

According to Ashok and Ben-Akiva [12], the structural deviation of OD flow, that is,  $X_h$ , follows a normal distribution. When the length of the time interval  $h$  is relatively short (e.g., 15 minutes in this study), it is reasonable to assume the structural deviations of OD flow between two consecutive intervals follows a random walk process as

$$X_h = X_{h-1} + W_h \tag{3}$$

where  $W_h$  represents the white noise. This makes it possible to process  $X_h$  as the state variable, so Equation (3) can be considered as the state equation of the Kalman filter model.

2.2.2. Establishment of the measurement equation

(a) OD flow and traffic volume relationship

Establishing a reasonable mapping relationship between traffic volumes and OD flows is crucial for dynamic OD estimation. Because the number of links and intersections with known traffic volumes is generally much smaller than the number of OD pairs, the OD estimation requires solving indeterminate equations. Given that most previous studies have used link volumes in their OD estimation, additional constraint equations could be added to the OD estimation process to improve its accuracy if turning movement volumes at intersections are available. Because traffic sensors are now widely used on

urban roads for both real-time traffic monitoring and traffic signal time optimization, this study opted to combine turning movement volumes and link traffic volumes to estimate OD flows by examining the OD flow-turning movement volume and OD flow-link volume relationships.

For a specific OD pair, vehicles departing from the origin can take any one of multiple paths to reach their destination. Conversely, the turning movement volume is the sum of all the paths passing over the point where the turning movement happens. Because of the way traffic flow evolves over time and space [26], turning movement volumes are affected not only by the current OD flows, but also by the OD flows in previous intervals. Suppose that the turning movement volume  $c$  in current time interval  $h$  is the summation of the OD assignments in the  $h^{th}, (h-1)^{th}, \dots, (h-p'+1)^{th}$  intervals, then the OD flow-turning movement volume relationship in interval  $h$  can be formulated as

$$z_{ch} = \sum_{p=h-p'+1}^h \sum_{r=1}^{n_{OD}} b_{ch}^{rp} T_{rp} + v_{ch} \quad (4)$$

where  $z_{ch}$  is the volume of turning movement  $c$  observed during interval  $h$  ( $c=1,2,\dots,n_c$ , and  $n_c$  is the total number of the turning movements),  $p$  is the time interval index,  $T_{rp}$  is the total number of vehicles departing from the origin of the  $r^{th}$  OD pair during interval  $p$ ,  $b_{ch}^{rp}$  is the fraction of the  $r^{th}$  OD flow that leaves its origin during interval  $p$  and passes the intersection during interval  $h$ ,  $p'$  is the number of intervals where traffic volume is affected by a specific OD flow, and  $v_{ch}$  is the random error. Usually,  $p'$  is proportional to the longest path travel time on the network.

Similarly, the link volume-OD flow relationship during interval  $h$  is formulated as

$$y_{lh} = \sum_{p=h-p'+1}^h \sum_{r=1}^{n_{OD}} a_{lh}^{rp} T_{rp} + w_{lh} \quad (5)$$

where  $y_{lh}$  is the observed volume of link  $l$  during interval  $h$  ( $l=1,2,\dots,n_l$ ),  $p$  is the time interval index,  $T_{rp}$  is the total number of vehicles departing from the origin of the  $r^{th}$  OD pair during interval  $p$ ,  $a_{lh}^{rp}$  is the fraction of the  $r^{th}$  OD flow during interval  $p$  to link  $l$  in interval  $h$ ,  $p'$  is the number of intervals in which traffic volume is affected by the  $r^{th}$  OD flow, and  $w_{lh}$  is the random error.

Equations (4) and (5) can be converted into matrix form, shown in Equations (6) and (7), respectively, as

$$z_h = \sum_{p=h-p'+1}^h b_h^p T_p + v_h^{turn} \quad (6)$$

$$y_h = \sum_{p=h-p'+1}^h a_h^p T_p + v_h^{link} \quad (7)$$

where  $z_h$  is a  $n_c \times 1$  vector composed by  $z_{ch}$ ,  $T_p$  is a matrix composed by  $T_{rp}$ ,  $b_h^p$  is a  $n_c \times n_{OD}$  assignment matrix of contributions  $T_p$  to  $z_h$ ,  $v_h^{turn}$  is a  $n_c \times 1$  vector of random errors,  $y_h$  is a  $n_l \times 1$  vector composed by  $y_{lh}$ ,  $a_h^p$  is a  $n_l \times n_{OD}$  assignment matrix of contributions  $T_p$  to  $y_h$ , and  $v_h^{link}$  is a  $n_l \times 1$  vector of random errors.

#### (b) Measurement equation

Let  $y^H$  and  $z^H$  denote the historical link volume and historical turning movement volume, respectively. Equations (6) and (7) can be rewritten in difference form:

$$z_h - z_h^H = \sum_{p=h-p'+1}^h b_h^p (T_p - T_p^H) + v_h^{turn} \quad (8)$$

$$y_h - y_h^H = \sum_{p=h-p'+1}^h a_h^p (T_p - T_p^H) + v_h^{link} \quad (9)$$

It is assumed that the relationships between  $z^H$ ,  $y^H$ , and  $T_p^H$  are free from errors. If this is not the case, the error terms  $v_h^{turn}$  and  $v_h^{link}$  will pick up additional components in Equations (8) and (9).

Denote  $y'_h = y_h - y_h^H$  and  $z'_h = z_h - z_h^H$ , then

$$z'_h = \sum_{p=h-p'+1}^h b_h^p T'_h + v_h^{turn} \tag{10}$$

$$y'_h = \sum_{p=h-p'+1}^h a_h^p T'_h + v_h^{link} \tag{11}$$

Equations (12) and (13) can be combined to form the measurement equation of the Kalman filter model as

$$Y_h = A_h X_h + V_h \tag{12}$$

where  $Y_h = \begin{bmatrix} y'_h \\ z'_h \end{bmatrix}$ ,  $Y_h$  is a  $(n_l + n_c) \times 1$  vector,  $A_h = \begin{bmatrix} a_h & a_{h-1} & \dots & a_{h-p'+1} \\ b_h & b_{h-1} & \dots & b_{h-p'+1} \end{bmatrix}$  is a  $p'$  by  $(n_l + n_c) \times n_{OD}$  matrix, and  $V_h = \begin{bmatrix} v_h^{turn} \\ v_h^{link} \end{bmatrix}$  is a  $(n_l + n_c) \times 1$  vector of random errors.

(c) Dynamic traffic assignment

The dynamic traffic assignment (DTA) matrix is critical for establishing the measurement equation and showing the quantitative relationship between the traffic volumes and OD flows. Two approaches, the analytical method [27] or the simulation-based method [23, 28], are generally used to derive the assignment matrix. The analytical method is generally not suitable for operational applications due to its calculation complexity. Comparatively, even though the simulation-based method can take traveler's route choice into account to reduce the calculation complexity, it is difficult to integrate the OD estimation method into a traffic simulator for real-time applications. In this study, a method for deriving the assignment matrix of turning movement is proposed based on the method by Cascetta *et al.* [29]. Unlike the traditional method [30], which uses stochastic travel times as route impedances, real-time detected travel times are employed to represent changing route impedances. To improve the practicability of the method, the procedure for determining the effective path sets can be simplified.

Let  $M$  be the effective path set for all the OD pairs on an urban network. Then,  $M = |M_1 \cup M_2 \cup \dots \cup M_{n_{OD}}|$ , where  $n_{OD}$  is the number of OD pairs, and  $M_r (r = 1, 2, \dots, n_{OD})$  is the effective path set for the  $r^{th}$  OD pair. Suppose an OD pair has  $m$  effective paths, then  $m \in M_r$ . Let  $b_{ch}^{mp}$  denote the proportion of the number of vehicles departing from the origin of the OD pair through the  $m^{th}$  path in set  $M_r$  during time interval  $p$  that is included in the total volume in turning movement  $c$  during interval  $h$ , and let  $\tilde{b}_{ch}^{rp}$  denote the fraction of the  $r^{th}$  OD flow that leaves its origin during interval  $p$  and passes the intersection during interval  $h$ . Then,

$$\tilde{b}_{ch}^{rp} = \sum_{m \in M_r} b_{ch}^{mp} F_{rh}^m \tag{13}$$

where  $F_{rh}^m$  is the route choice rate, that is, the ratio of traffic flow to the total demand for OD pair  $r$  during interval  $h$  along path  $m$ .

Generally, travelers select their favorable routes according to route impedance. The method by Cascetta *et al.* [29] considers the value above a certain percentage of the shortest route to be the maximum acceptable impedance and regards the routes that have smaller impedances as effective routes. Even though effective routes are feasible route options on an urban network, it is assumed

that most travelers are more inclined to choose the shortest route. Let  $M_r$  denote the effective path set, then a Logit model can be used to calculate the rates of the effective routes chosen by travelers as

$$F_{rh}^m = \frac{\exp\left[-TT_{rh}^m/\bar{TT}_{rh}\right]}{\sum_{z \in M_r} \exp\left[-TT_{rh}^z/\bar{TT}_{rh}\right]} \tag{14}$$

where  $F_{rh}^m$  is the ratio of travelers that choose path  $m$  for the  $r^{th}$  OD pair during interval  $h$ ,  $TT_{rh}^m$  is the impedance of path  $m$  for the  $r^{th}$  OD pair during interval  $h$ , and  $\bar{TT}_{rh}$  is the average impedance of all the paths for the  $r^{th}$  OD pair during interval  $h$ . In this study, the impedances are considered to be the travel times, calculated as the sum of the link (defined as the road section between two consecutive intersections) travel times plus the average traffic signal control delays.

Suppose all vehicles traveling along path  $m$  during interval  $p$  travel within the envelope of a single packet  $(m, p)$ , then the movement of all those vehicles will be subject to a homogeneous distribution. Considering the time a vehicle reaches the intersection as the moment when the turning movement occurs, the times the first and last vehicle in packet  $(m, p)$  complete the turning movement  $c$  are denoted as  $\eta_{1c}^{mp}$  and  $\eta_{2c}^{mp}$ , respectively; thus, the time interval during which all the vehicles in packet  $(m, p)$  complete turning movement  $c$  is  $\Delta_{cm} = [\eta_{1c}^{mp}, \eta_{2c}^{mp}]$ , and  $b_{ch}^{mp}$  can be calculated as

$$b_{ch}^{mp} = \frac{|\Delta_{cm} \cap \Delta_h|}{|\Delta_{cm}|} \tag{15}$$

where  $\Delta_h$  is the length range of interval  $h$ , and  $|\Delta_{cm} \cap \Delta_h|$  is the intersection length of two intervals  $\Delta_{cm}$  and  $\Delta_h$ .

### 2.3. Model solution

The dual-adaptive mechanism proposed in this study consists of two parts. The first estimates the structural deviation of the OD flow based on the Kalman filter model and updates the filter parameters iteratively, whereas the other updates the regular OD using estimated real-time OD flows.

Let  $Q_h$  and  $R_h$  denote the covariance matrices of dynamic noises of  $W_h$  and  $V_h$ , respectively. A set of regular time-dependent ODs must be initialized as the seed of a DTA mechanism using historical data, after which the initial state of  $X_0$ ,  $P_h$ ,  $Q_h$ , and  $R_h$  can be calibrated. Based on this, the Kalman filter model can then be solved using the well-known time-update recursions [31] as follows.

Step 1: Priori estimate of the state vector

$$X_{h|h-1} = X_{h-1} \tag{16}$$

Step 2: Priori estimate of the state error covariance

$$P_{h|h-1} = P_{h-1} + Q_{h-1} \tag{17}$$

Step 3: Calculate measurement noise with measurement

$$V_h = Y_h - A_h X_{h|h-1} \tag{18}$$

Step 4: Calculate measurement noise covariance

$$R_h = \frac{1}{n-1} \sum_{i=1}^n \left\{ (V_i - \bar{V})(V_i - \bar{V})^T - \frac{n-1}{n} X_i P_{i|i-1} X_i^T \right\} \tag{19}$$

Step 5: Compute Kalman gain

$$K_h = P_{h|h-1} A_h^T (A_h P_{h|h-1} A_h^T + R_h)^{-1} \quad (20)$$

Step 6: Posterior estimate of state vector

$$X_h = X_{h|h-1} + K_h V_h \quad (21)$$

Step 7: Posterior estimate of state error covariance

$$P_{h+1} = (I - K_{h+1} A_{h+1}) P_{h+1|h} \quad (22)$$

Step 8: Compute the measurement error

$$W_h = X_{h|h-1} - X_{h-1} \quad (23)$$

Step 9: Update measurement noise covariance

$$Q_h = \frac{1}{n-1} \sum_{i=1}^n \left\{ (W_i - \bar{W})(W_i - \bar{W})^T - \frac{n-1}{n} (P_{i-1} - P_i) \right\} \quad (24)$$

Note that  $n$  is a prescribed parameter indicating the memory size for updating  $Q_h$  and  $R_h$ . Theoretically, the OD deviation series  $\{X_h\}$  must be a stationary series to ensure convergence, and the speed of the convergence relies on model initialization. The accuracy of the initial parameters also affects the accuracy of the model solutions. In practice, this suggests that one can refer to the historical OD flows to decide the initial values of the parameters.

Based on the dynamic estimates of the structural deviation of OD flows, the regular ODs for the time-of-day and day-of-week can be updated on a daily basis, as shown in Figure 1. The formula for the updating process is

$$T_h^H(d) = \alpha T_h^H(d-7) + (1-\alpha) T_h(d) \quad (25)$$

where  $\alpha$  is an adaptive coefficient,  $T_h^H(d-7)$  and  $T_h^H(d)$  indicate the regular OD for day  $d-7$  and day  $d$  during interval  $h$ , and  $T_h(d-7)$  is the time-dependent OD for day  $d-7$  during interval  $h$ .

### 3. IMPLEMENTATION

#### 3.1. Data collection

To validate the proposed method, the urban road network in the downtown area of the city of Kunshan in China was selected. The road network is composed of nine major arterials, three secondary roads, six collector roads, and 58 intersections (48 signalized intersections). On the road network, microwave vehicle detectors are installed to collect link traffic volumes, and video vehicle detectors are installed to collect turning movement volumes at intersections. Figure 2 shows the study area of the urban traffic network and the locations of all traffic sensors.

On the road network, 156 links (two directions) capture detected traffic volumes and 46 intersections provide lane by lane volumes at the approach to each intersection. However, because of existing mixed-used lanes at intersections, only 67 turning movement volumes were collected. The measurements, including volume and speed, were collected for the period from September 3, 2012 to September 30, 2012 and were aggregated at 15-minute intervals. The data collected from September 3 to September 16 were

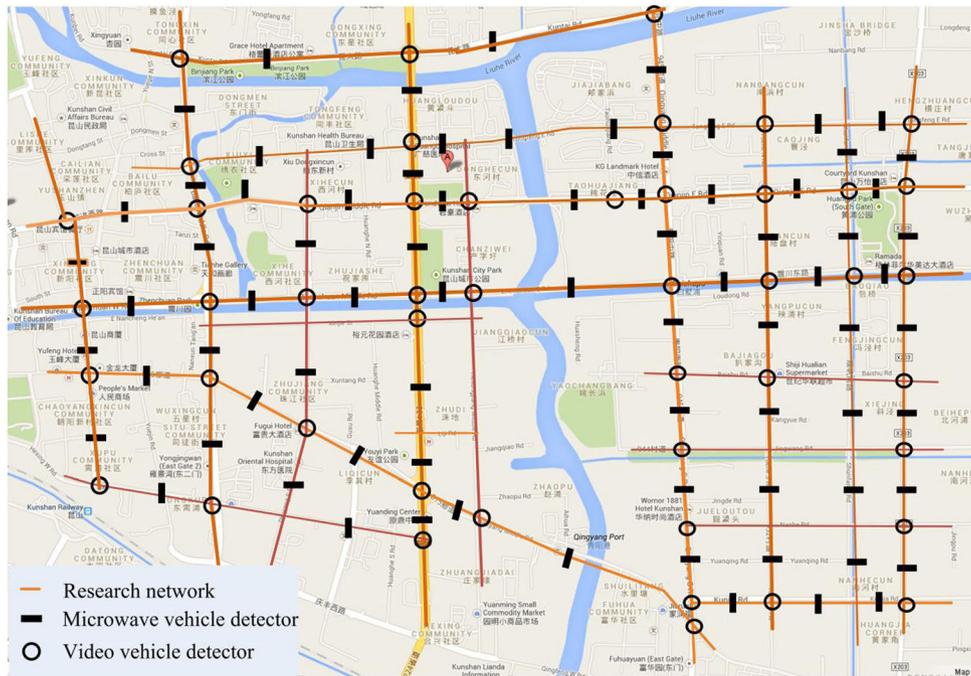


Figure 2. Study area and traffic sensor locations.

used as the calibration dataset and the data collected from September 17 to September 30 were used for model validation. Note that the traffic measurements had passed traditional quality screening tests and had been used to perform data imputation using historical average methods prior to their use in this work.

### 3.2. Initialization of regular ODs

In order to initialize the regular ODs, road network modeling was conducted using Q-PARAMICS. Figure 3 shows the topological graph of the study network. Using the 2 weeks of traffic data from the calibration dataset, both the link and turning movement volumes were averaged by time-of-day and day-of-week, then a set of regular OD flows were initialized as the OD estimates from the PARAMICS simulation using the averaged link and turning movement volumes as input data. In practice, the greater the variation in the OD pattern observed, the lower the  $\alpha$  value considered. Based on the limited regular ODs initialized in this study,  $\alpha$  is selected as 0.9 that results in the smallest mean square error (MSE). Given the initialized regular ODs  $T_h$  with observations  $T_1, T_2, \dots, T_n$ , the mean

square error is calculated as 
$$\frac{\sum_{h=1}^n (T_h^H - T_h)^2}{n}.$$

### 3.3. Dynamic traffic assignment

In this study, the assignment matrices were iteratively calculated using the link travel times and intersection control delays as the route impedance. The dynamic OD flows were then estimated by the proposed Kalman filter model and the estimated link and turning movement volumes were calculated.

To evaluate the DTA performance, statistical indicators such as mean absolute percentage error (MAPE) and root mean square error (RMSE) can be calculated to compare observed and estimated traffic volumes. In this study, because the traffic volumes varied significantly over the different urban road types in this real road network, the GEH statistic [32] that is widely used in traffic forecasting and traffic modeling was applied to avoid some of the pitfalls that typically occur when using simple percentages to compare two sets of volumes. GEH, which was invented by Geoffrey E. Havers, takes its name from his initials. The GEH can be calculated as

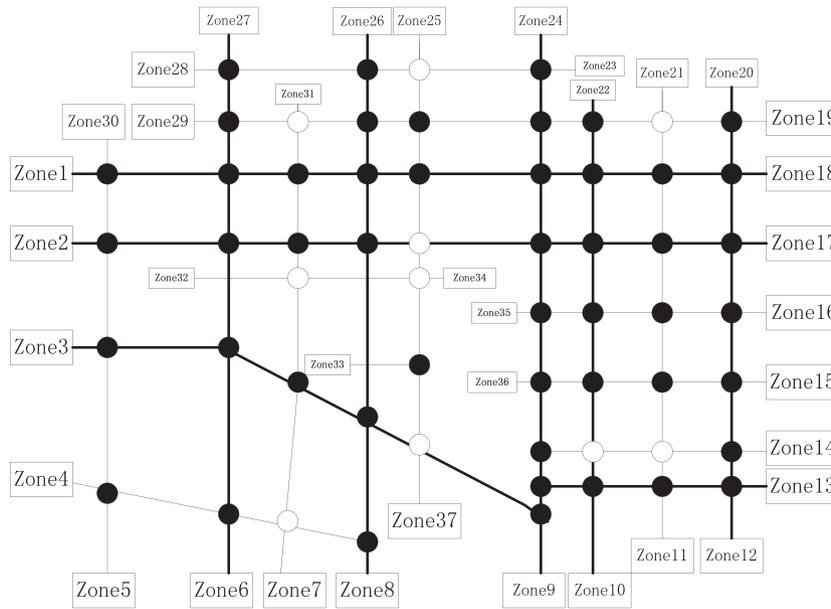


Figure 3. Topological graph of the study network.

$$GEH = \sqrt{\frac{(V_m - V_c)^2}{(V_m + V_c)/2}} \tag{26}$$

where  $V_m$  is the observed traffic flow rate (veh/hour) and  $V_c$  is the estimated traffic volume (veh/hour). If GEH is smaller than 5.0, the difference between  $V_m$  and  $V_c$  is considered acceptable. If 85% of the GEH values are lower than 5.0, the model is considered to perform well [33].

Figure 4 shows the average GEH values at different time intervals for all traffic collection stations during the period 6:45 AM to 8:00 PM on September 24, 2012. As the figure shows, 87% (47 out of 54) of the GEH values are lower than 5, indicating that the proposed traffic assignment method performs well and that the estimated volumes closely match the observed volumes. Those occasions where  $GEH > 5$  generally occurred were times during the morning (7:00 AM–9:00 AM) and evening (4:30 PM–6:30 PM) peak hours, indicating that the accuracy of the estimated volumes during peak hours is lower than that during other periods. This is probably because traffic congestion creates more uncertainties in route choices. Additionally, the queues at intersections make it difficult for most traffic assignment models, including this one, to correctly model drivers’ complex decision process during peak hours.

Figure 5 compares the estimated and observed volumes for several specific links and turning movements. As the figure shows, the estimated volumes are close to the observed volumes for all the periods and the volumes follow the same general trend. In Figures 6(a) and (b), the estimated and observed volumes match closely during non-peak hours. Larger offsets are found during peak hours, but the GEH values are still within the acceptable range (e.g.,  $GEH = 3.54$  for the left turning movement at Intersection 1032 at 5:30 PM), and the largest volume offset happens at 5:30 PM in both Figures 6(a)

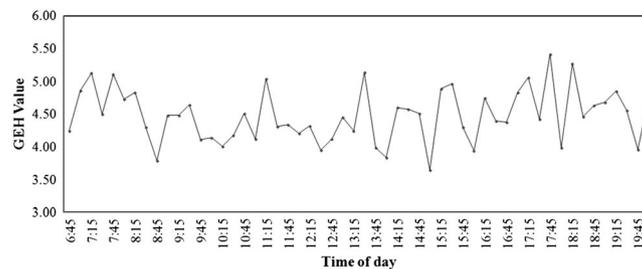


Figure 4. Average GEH value of all traffic counts in different periods.

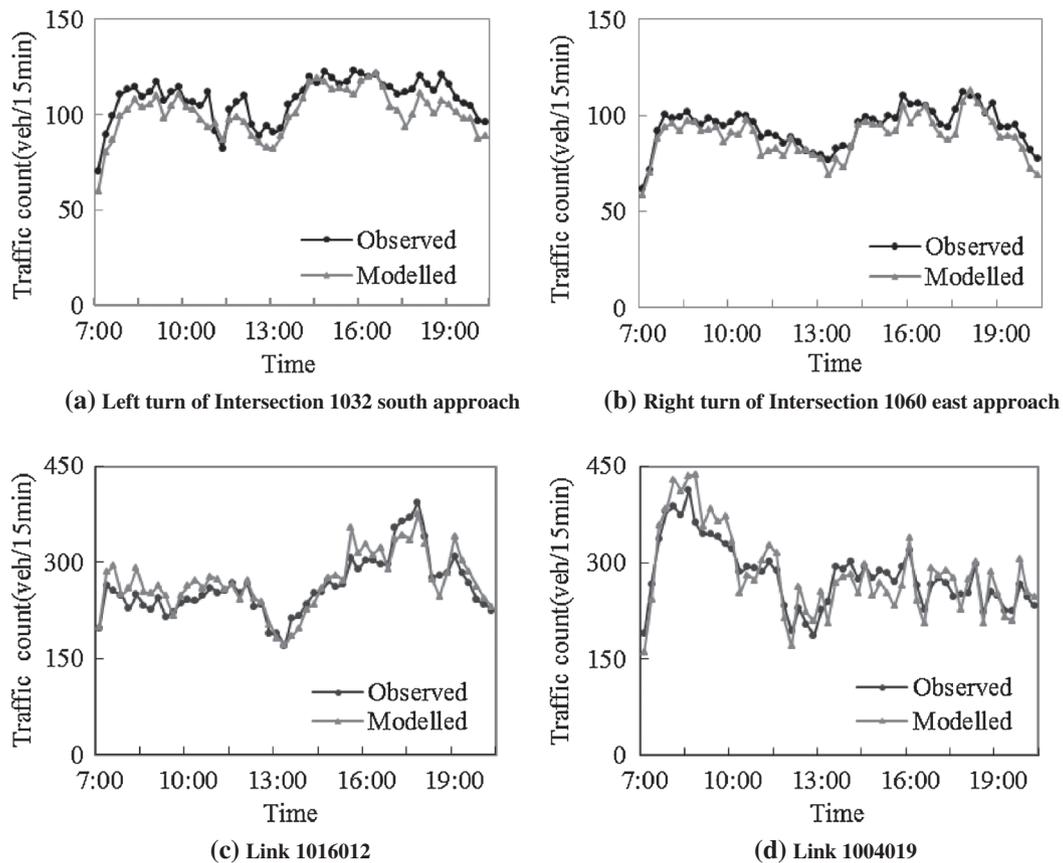


Figure 5. Comparison of the estimated and observed volumes of the example links and turning movements.

(offset = 18) and 6(b) (offset = 13), with GEH values of 1.77 and 2.67, respectively. Even though the offsets are larger during peak hours, the low GEH values show the accuracy of the proposed method. Comparing the link and turning movement volumes, one can see that the fluctuation of the turning movement volumes is smaller than that of link volumes, indicating that the estimation results for the turning movement volumes are more accurate than those for the link volumes.

#### 4. PERFORMANCE EVALUATION

##### 4.1. Overall performance

The overall performance of the proposed model was evaluated by calculating the mean absolute error (MAE), MAPE, and RMSE between the estimated ODs and the observed ODs. The observed time-dependent ODs were surveyed by matching license plates captured by video cameras. The selected road network based on the topological graph of the study network generated a total of 1332 OD pairs, among which 1249 OD pairs generally have no demand or volumes of fewer than 10 vehicles every 15 minutes. After removing these OD flows, 21 typical OD pairs were selected and evaluated. The results of the 21 typical OD pairs are given in Table I.

As the table shows, the MAEs between the estimated and observed values were all less than 25 vehicles per 15 minutes, but the MAPEs varied between 7.58% and 24.48%. It is important to note, however, that even a small error may result in a large MAPE (e.g., OD pairs 2–13 and 19–3) when the average OD flow is small. The OD pairs experiencing heavy demand (e.g., OD pairs 2–17 and 20–12) had MAPEs below 13%, with some even falling below 10%. Overall, the proposed method performs well and can produce satisfactory OD estimates.

To go further in evaluating the performance of the proposed method, several typical OD pairs were selected and their estimated and observed volumes were compared. Figure 6 shows two examples of

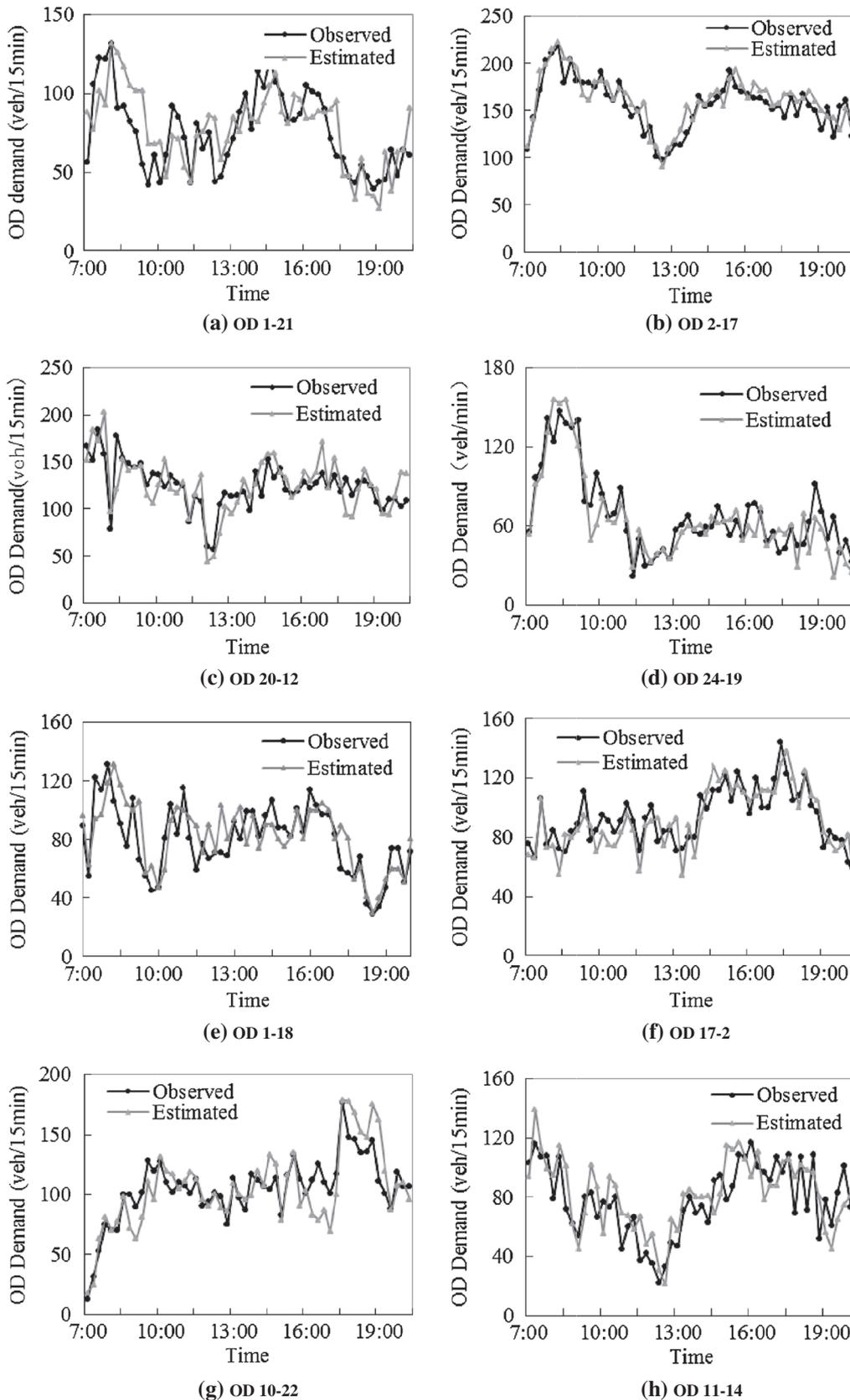


Figure 6. Estimated and observed origin-destination flows: (a)–(d) are for September 25, 2012 (Monday) and (e)–(h) are for September 30, 2012 (Sunday).

Table I. Evaluation results for the proposed method.

OD pairs	Average OD* (veh/15 minutes)	MAE (veh/15 minutes)	MAPE (%)	RMSE (veh/15 minutes)
1-21	76	16	24.06	20
2-13	32	7	24.48	7
3-6	35	5	18.52	6
2-17	156	11	7.58	14
6-29	68	10	16.26	14
7-9	50	9	21.73	11
8-34	54	11	22.81	15
9-13	53	10	19.28	13
10-22	105	13	13.03	18
11-14	78	15	22.58	17
12-19	179	24	13.33	29
14-17	56	11	22.56	13
17-2	93	10	11.06	11
18-24	145	12	8.60	15
17-20	91	14	22.48	19
18-14	74	12	17.29	15
19-3	21	3	17.41	4
20-12	124	16	12.76	20
24-19	68	11	17.04	15
24-23	97	12	12.48	17
22-10	102	14	13.85	17

OD, origin-destination; MAE, mean absolute error; MAPE, mean absolute percentage error; RMSE, root mean square error.

\*Average OD = the average value of the specific observed OD pair over a single day.

Table II. Evaluated results of three methods.

OD pairs	Estimation method								
	Proposed method			Link-volume-based			Turning-movement-based		
	MAE (veh/15 minutes)	MAPE (%)	RMSE (veh/15 minutes)	MAE (veh/15 minutes)	MAPE (%)	RMSE (veh/15 minutes)	MAE (veh/15 minutes)	MAPE (%)	RMSE (veh/15 minutes)
1-21	16	24.06	20	19	27.91	24	22	31.89	27
2-13	7	24.48	7	10	32.59	13	11	38.43	13
3-6	5	18.52	6	9	26.07	11	11	31.50	13
2-17	11	7.58	14	13	8.52	15	18	11.80	23
6-29	10	16.26	14	17	26.34	22	19	31.87	24
7-9	9	21.73	11	11	31.28	14	10	36.86	13
8-34	11	22.81	15	13	24.97	16	14	27.71	17
9-13	10	19.28	13	12	24.58	15	19	32.89	25
10-22	13	13.03	18	16	15.16	21	33	33.06	42
11-14	15	22.58	17	17	26.70	21	20	31.10	24
12-19	24	13.33	29	28	15.65	33	40	21.90	51
14-17	11	22.56	13	13	26.77	16	16	30.22	19
17-2	10	11.06	11	13	14.52	15	20	22.14	25
18-24	12	8.60	15	15	11.13	18	24	17.50	29
17-20	14	22.48	19	20	28.43	24	23	31.15	29
18-14	12	17.29	15	16	21.08	18	21	30.11	27
19-3	3	17.41	4	4	24.65	5	4	29.96	5
20-12	16	12.76	20	23	19.12	27	34	27.61	43
24-19	11	17.04	15	14	24.19	17	18	30.64	21
24-23	12	12.48	17	16	17.22	20	32	33.20	39
22-10	14	13.85	17	15	14.34	19	27	26.09	33
Average	12	17.42	15	15	22.43	18	21	29.28	26

OD, origin-destination; MAE, mean absolute error; MAPE, mean absolute percentage error; RMSE, root mean square error.

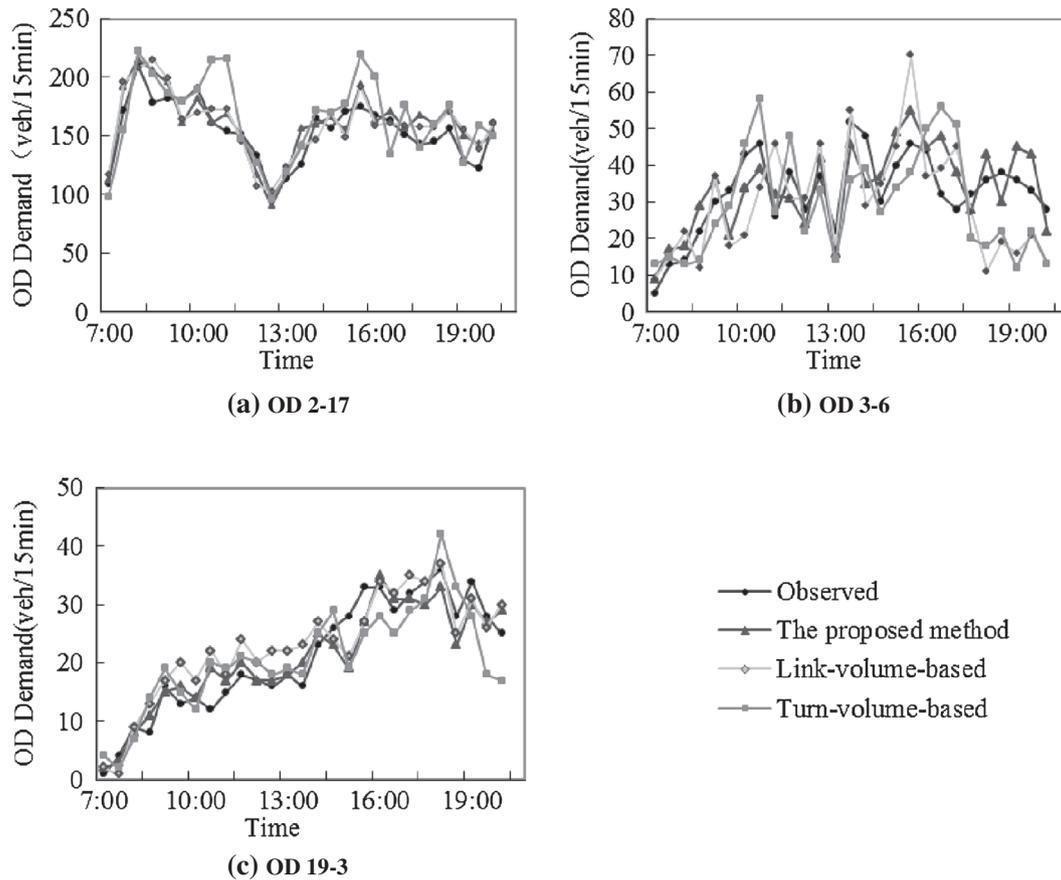


Figure 7. Comparison of the performance of the three methods.

the OD flow comparisons on September 25, 2012 (Monday) and September 30, 2012 (Sunday). As the figure shows, the proposed model achieved a satisfactory performance for both weekday and weekend levels of road use. These results show that the proposed method can accommodate a wide range of OD flows as well as changes in traffic patterns. Because of the disturbance of the input/output traffic flows from the secondary and collector roads, the estimated results of several OD pairs—for example, those shown in Figures 6(b) and (c)—show that estimated volumes for a destination on arterial roads are more accurate than those on secondary or collector roads (Figures 6(a) and (h)). As the data in Figures 6(d) and (g) demonstrate, the estimated volumes during afternoon peak hours (4:30 PM–6:30 PM) were less accurate than those during other time periods. As mentioned earlier, the calculation accuracy is inevitably affected by traffic congestion and queuing effects during peak hours.

#### 4.2. Comparative evaluation

To evaluate the benefits of multi-source data for OD estimation, the proposed method was compared with the link-volume-based method (using link volume only) and the turning-movement-based method (using the turning movement volume only) for observed OD flows. The results for the 21 OD pairs are shown in Table II.

Table II shows that the average MAEs, MAPEs, and RMSEs of the proposed method are consistently smaller than those of either of the other two methods. The proposed method clearly outperforms both the link-volume-based method and the turning-movement-based method. As expected, the results show that the proposed method fully utilizes multisource sensor data to improve the accuracy of OD estimation. Interestingly, the link-volume-based method slightly outperforms the turning-movement-based method here. The relatively high MAE and MAPE produced by the turning-movement-based method might be because only 67 turning movement volumes were available, which is not sufficient to effectively capture changing trends in OD flows.

Figure 7 shows the estimated and observed values of three OD pairs obtained by using each of the three methods and compares the results. These results show that the proposed method clearly fits the observed values more closely than either of the other two methods. As shown in Figures 7(b) and (c), results of the link-volume-based and the turning-movement-based methods deviate considerably from the observed values during all time periods, especially during the afternoon peak hours (after 4:30 PM). This comparison demonstrates the benefits gained by using multisensor data to smooth the fluctuations that occur with single source data and thus improve the stability of the estimated results.

## 5. CONCLUSIONS

This paper proposes a Kalman filter approach to dynamic OD estimation using multisource sensor data. Based on the framework of the proposed method, the dynamic changes over time of the characteristics of the OD flow are analyzed, and the problem of dynamic OD estimation is converted to a problem of estimating the OD structural deviation. The dynamic relationship between traffic volume and OD structure deviation was then utilized to establish a Kalman filter model for iterative OD estimation, in which an improved traffic assignment approach was embedded to enable the traffic assignment matrix to be updated in real time. Finally, a dual self-adaptive mechanism based on the Kalman filter was used to calibrate the model, estimate the OD structural deviation, and update the regular OD.

The proposed method was implemented on a real-life traffic network in the downtown area of Kunshan City, China, in which the regular OD flows were initialized as the OD estimates from the PARAMICS simulation using a calibration dataset. The results of the implementation revealed that the proposed method performs well in actual traffic scenarios. The traffic volumes estimated by the proposed traffic assignment approach closely matched the observed volumes and met the general requirement of the dynamic OD estimation process. The results of the OD estimation represented variations in real-time OD flow well and the errors fell within an acceptable range. Comparative tests revealed that the proposed method outperformed both the link-volume-based and turning-movement-based methods.

The proposed method offers an effective way to utilize multi-source sensor data to achieve high-accuracy time-dependent traffic assignment and dynamic OD estimation. However, despite these encouraging results, the proposed model suffers from a number of limitations that should be addressed in future work. First, the initialization of the proposed method relies on other OD estimation tools such as PARAMICS, which can be time-consuming. One potential solution to this problem is to integrate the initialization process into the OD estimation method, so as to initialize the model automatically. Second, the dynamic assignment method may not be able to maintain high accuracy for periods that experience serious traffic congestion, especially in a large network. One possible way to deal with this could be to use a simulation-based method to calculate the dynamic assignment matrices.

Recent advances in traffic detection technologies may open new avenues for alternative data collection methods that could be considered in future applications. These could include, for instance, floating car, Bluetooth-based travel time collection [34], and/or Radio Frequency Identification techniques. New traffic sensors will make it possible for large amounts of data to be collected. The advantage of gathering 'big data' is that traffic conditions can be more accurately identified, whereas the disadvantage is that some additional processes may be required to remove invalid and redundant data from the data pool and may thus make it challenging to use 'big data' to estimate the dynamic ODs. In terms of future work, the proposed method can be further revised based on the data collected by multiple sources of advanced traffic sensors.

## 6. LIST OF SYMBOLS AND ABBREVIATIONS

### 6.1. Symbols

$a_h^p$	a $n_l \times n_{OD}$ assignment matrix of contributions $T_p$ to $y_h$
$a_{lh}^{rp}$	the fraction of the $r^{\text{th}}$ OD flow during interval $p$ to the volume of link $l$ in interval $h$
$A_h$	is equal to $\begin{bmatrix} a_h & a_{h-1} & \dots & a_{h-p'+1} \\ b_h & b_{h-1} & \dots & b_{h-p'+1} \end{bmatrix}$

$b_h^p$	a $n_c \times n_{OD}$ assignment matrix of contributions $T_p$ to $z_h$
$b_{ch}^{mp}$	the fraction of the OD flow that leaves its origin through the $m^{th}$ path in set $M_r$ during interval $p$ and passes the intersection $c$ during interval $h$
$\tilde{b}_{ch}^{rp}$	the fraction of the $r^{th}$ OD flow that leaves its origin during interval $p$ and passes the intersection $c$ during interval $h$
$b_{ch}^{rp}$	the fraction of the $r^{th}$ OD flow that leaves its origin during interval $p$ and passes the intersection $c$ during interval $h$
$c$	the $c^{th}$ turning movements
$F_{rh}^m$	the ratio of traffic flow to the total demand for OD pair $r$ during interval $h$ along path $m$
$F_{rh}^m$	the ratio of travelers that choose path $m$ for the $r^{th}$ OD pair during interval $h$
$h$	the $h^{th}$ time interval
$l$	the link index
$m$	the index of the effective path, $m \in M_r$
$M$	the effective path set for all the OD pairs
$M_r$	the effective path set for the $r^{th}$ OD pair
$n_{OD}$	the total number of OD pairs
$n_c$	the total number of the turning movements
$n_l$	the total number of the links
$p$	the time interval index
$p'$	the number of intervals in which traffic volume is affected by a specific OD flow
$R_h$	the covariance matrices of dynamic noises of $V_h$
$Q_h$	the covariance matrices of dynamic noises of $W_h$
$T_h$	the dynamic OD flows in time interval $h$
$T_h^H$	the regular OD flows in time interval $h$
$T_p$	a matrix composed by $T_{rp}$
$T_{rp}$	the total number of vehicles departing from the origin of the $r^{th}$ OD pair during interval $p$
$T_h^H(d-7)$	the regular OD for day $d-7$ during interval $h$
$T_h^H(d)$	the regular OD for day $d$ during interval $h$
$T_h(d)$	the time-dependent OD for day $d$ during interval $h$
$TT_{rh}^m$	the impedance of path $m$ for the $r^{th}$ OD pair during interval $h$
$TT_{rh}^-$	the average impedance of all the paths for the $r^{th}$ OD pair during interval $h$
$v_{ch}$	the random error of the turning movements $c$ in interval $h$
$v_h^{turn}$	a $n_c \times 1$ vector of random errors of turning movement volume in interval $h$
$v_h^{link}$	a $n_l \times 1$ vector of random errors of turning movement volume in interval $h$
$V_h$	is equal to $\begin{bmatrix} v_h^{turn} \\ v_h^{link} \end{bmatrix}$
$V_m$	the observed traffic flow rate (veh/hour)
$V_c$	the estimated traffic volume (veh/hour)
$W_h$	the white noise of the link volumes in time interval $h$
$w_{lh}$	the random error of the volume in link $l$ in interval $h$
$X_h$	the structural deviation of the OD flow in time interval $h$
$y_{lh}$	the observed volume of link $l$ during interval $h$
$y_h$	a $n_l \times 1$ vector composed by $y_{lh}$
$y_h^H$	the historical link volume
$y_h'$	the difference between $y_h$ and $y_h^H$
$Y_h$	is equal to $\begin{bmatrix} y_h' \\ z_h' \end{bmatrix}$
$z_h$	a $n_c \times 1$ vector composed by $z_{ch}$
$z_h^H$	the volume of historical turning movement
$z_h'$	the difference between $z_h$ and $z_h^H$
$z_{ch}$	the volume of turning movement $c$ observed during interval $h$
$\alpha$	the adaptive coefficient

$\varepsilon_h$	the random error in time interval $h$
$\eta_{1c}^{mp}$	the times the first vehicle in packet $(m, p)$ complete the turning movement $c$
$\eta_{2c}^{mp}$	the times the last vehicle in packet $(m, p)$ complete the turning movement $c$
$\Delta_{cm}$	the length range of the time interval during which all the vehicles in packet $(m, p)$ pass turning movement $c$
$\Delta_h$	the length range of interval $h$

## 6.2. Abbreviations

AM	ante meridiem
DTA	dynamic traffic assignment
OD	origin-destination
GEH	Geoffrey E. Havers
MSE	mean square error
MAE	mean absolute error
MAPE	mean absolute percentage error
PM	post meridiem
Q-PARAMICS	Quadstone PARAMICS software
RMSE	root mean square error

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