Travel Time Reliability Using the Hasofer–Lind–Rackwitz–Fiessler Algorithm and Kernel Density Estimation

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Travel time reliability has emerged as an indicator of roadway performance. Estimation of travel time distribution is an important starting input for measuring travel time reliability. This study used kernel density estimation to estimate travel time distribution. The Hasofer–Lind–Rackwitz–Fiessler algorithm, widely used in the field of reliability engineering, was used in this work to compute the reliability index of a system based on its previous performance. The computing procedure for travel time reliability of corridors on a freeway was introduced, then network travel time reliability was developed. Given probability distributions estimated by the kernel density estimation technique and an anticipated travel time from travelers, the two equations of corridor and network travel time reliability can be used to address the question, "How reliable is my perceived travel time?" The definition of travel time reliability was in the sense of on-time performance, and this study was conducted from the perspective of travelers. The major advantages of the proposed method are as follows: (*a*) it demonstrates an alternative way to estimate travel time distributions when the choice of probability distribution family is still uncertain and (*b*) it shows its flexibility for application to levels of roadways (e.g., individual roadway segment or network). A user-defined anticipated travel time can be input, and travelers can use the computed travel time reliability information to plan their trips so that they can better manage trip time, reduce costs, and avoid frustration.

Travel time is one of the important freeway and arterial performance measures. In the past decade, many methods have been developed to estimate travel times (*1, 2*). Since travel time estimation methods are still maturing, however, travel time predictions might not accurately represent, or align with, road users' experiences on freeways. A 10-min drive from Point A to Point B, for example, might result in different levels of satisfaction, depending on time of day or specific expectations of road users. Recently, travel time reliability has emerged as another indicator of roadway performance. A Google Scholar search revealed that the number of research projects and papers on travel time reliability has been growing, from 2,430 in 2001 to 5,280 in 2012. Chen et al. stated that along with conventional freeway performance measures, such as level of service, vehicle miles traveled, and total delay, travel time reliability can perform as a major indicator of service quality for travelers and can be used to quantify travel cost for individual trips (*3*). Travel costs increase as either travel time increases or travel time reliability decreases. Van Lint et al. stated that travelers are inclined to choose more reliable routes instead of (on average) faster ones (*4*). Therefore, travel time reliability could be one of the factors that greatly affect transportation mode choice and route choice for individual travelers.

In addition to being a performance measure of traffic operations, travel time reliability is being used by regional transportation planning organizations to improve planning and operations at a macroscopic level (*5*). Organizations have begun to use travel time reliability as a primary measure of roadway congestion, instead of conventional measures such as volume-to-capacity ratio. Use of travel time reliability may be a more suitable approach for measuring changes of a transportation system. Although travel time reliability has been widely used by transportation planning and operations organizations, the definitions of travel time reliability vary depending on the purpose of the applications. FHWA officially defines travel time reliability as "the consistency or dependability in travel times, as measured from day-to-day and/or across different times of the day" (*6*). From the traveler's standpoint, this term could be interpreted as, how reliable is the anticipated travel time for my planned trip? or, in nine out of 10 trips, could I arrive at my planned destination within my anticipated time (90% reliability)? This definition is built on the concept of on-time performance. van Lint and van Zuylen stated that "travel time reliability relates to properties of the (day-to-day) travel time distribution as a function of time of day (TOD), day of the week (DOW), and month of year (MOY), as well as external factors such as weather, incidents, and road work" (*7*). This statement reveals that travel time reliability contains two components: selection of time periods and external factors. The capacity (supply) of transportation infrastructure can be temporarily reduced by external factors, such as weather, vehicle crashes, work zones, and geometric design; traffic demand can be affected by other external factors, such as special events. These external factors could affect travel time reliability (*8, 9*).

A considerable amount of research has focused on quantifying travel time reliability from various perspectives. Many current measures were based on mean or percentiles of travel time, such as 90th, 95th, or another percentile travel time, the buffer index, buffer time, the planning time index, the misery index, and the Florida reliability method. These measures were well defined and rephrased in published reports (*6, 10, 11*) and papers (*4, 12–15*). Practitioners preferred these travel time reliability measures because they were

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estimated either in minutes (the total travel time or extra time needed to reach destinations) or in percentages (level of on-time travel performance). From the reliability engineering point of view, Emam and Al-Deek used real-life traffic data to test four travel time distributions and found the lognormal distribution fit the data best (*13*). On the basis of the lognormal distribution, they proposed a new method for computing on-time performance for freeway corridors. However, Emam and Al-Deek made an assumption that the relationship between consecutive links of freeways was independent (*13*). The corridor travel time reliability was equal to the products of the reliability of each link. The drawback of their method is that the results of corridor travel time reliability may be close to zero as the number of links increases (*13*).

Unlike the measures generated from the practitioner perspective, many measures have been derived from statistics of the travel time distribution, such as standard deviation and coefficient of variation of travel times. van Lint and van Zuylen (*7*) and van Lint et al. (*4*) proposed skewness (λ_{skew}) and width (λ_{var}) of the estimated travel time distribution to represent the travel time reliability, and they visualized the proposed measures with a travel time reliability map. Rakha et al. proposed five methods for estimating travel time variance as measures of travel time reliability on the premise that the travel time distribution followed the lognormal distribution (*16*).

The distribution of travel time is believed to be an important starting input for measuring travel time reliability. The shape of travel time distribution relies heavily on traffic flow conditions. Van Lint and van Zuylen (*7*) and van Lint et al. (*4*) described travel time distributions according to four stages of traffic flow: free-flow condition, congestion onset, congestion, and congestion dissolve. Guo et al. used those results to restate the strong connection between travel time distributions and traffic flow conditions (*17, 18*). This connection was also reconfirmed by Pu, who investigated the analytic relationships between different travel time reliability measures (*19*).

Most previous travel time reliability research focused on the properties of the travel time distribution before developing measures of travel time reliability. Several studies applied simulation methods to construct the travel time distribution (*17, 20*). Most of the relevant studies fit statistical distribution models by using real-life traffic data (*12, 13, 21*). One travel time reliability measure, travel time window, was derived with a normal distribution (*10*). Skewed statistical distributions are most commonly found in previous research. Polus claimed that the gamma distribution could best be fit by the travel time data collected from arterial roads (*21*). Al-Deek and Emam conducted travel time reliability research in a transportation network environment and found the Weibull distribution could be representative of travel time distribution, rather than the exponential distribution (*12*). Many studies concluded that the lognormal distribution outperformed other skewed distributions in various traffic flow conditions (*9, 13, 16*). The lognormal distribution was therefore adopted in relevant research to investigate the analytic relationship of travel time reliability measures (*19*) or to develop a new measure (*14*). These researchers used a single model to represent the travel time distribution in a given period. Guo et al. stated that a mixture model outperformed single models, especially in a traffic congestion condition (*17*). They first proposed a two-state model based on the normal distribution and calibrated the required parameters of the two-state model (*18*). Later, Guo et al. replaced the normal distribution in the two-state model with a skewed distribution, namely, the lognormal distribution, and concluded that the skewed mixture model performed the best during peak hours (*22*).

The probability distributions referred to are parametric probability distributions, meaning that parameters are required to determine the locations and shapes for the predefined distribution types. In most cases, however, the traffic flow condition in a given period is unknown, which leads to difficulty in identifying an appropriate parametric probability distribution. Moreover, the parametric probability distributions may not be sufficiently adaptable when researchers are building travel time distributions. Nonparametric probability distributions may instead be preferred as appropriate statistical models for constructing travel time distributions because fewer assumptions are required to build them. The resulting nonparametric distributions can then be used for general cases in which significant flexibility is important.

This paper uses the kernel density estimation (KDE) technique to construct travel time distributions with greater flexibility, increased fidelity, and fewer assumptions. Moreover, since previous work focused on link or corridor travel time reliability, a framework based on the Hasofer–Lind–Rackwitz–Fiessler (HL-RF) algorithm that considers multilevel (e.g., corridor or network) cases for reporting travel time reliability is proposed in the paper. The methods for estimating travel time distribution and reliability with KDE and the HL-RF algorithm used in this study can be considered a general approach to constructing the travel time distributions and reliability. The impacts of external factors are not specifically discussed in this study.

The outline of the paper is as follows. The modeling framework is introduced first. This framework consists of three parts: travel time estimation, travel time distribution estimation, and travel time reliability index calculation. A description of the data set used in this study is followed by the implementation and applications of the proposed method. The paper ends with a concluding discussion and remarks.

MODELING FRAMEWORK

The proposed travel time reliability modeling framework consists of three parts: travel time calculation, link travel time distribution estimation, and corridor–network travel time reliability calculation. Travel time distribution is determined by the results of travel time estimation on a specific road segment at a given time period.

Travel Time Estimation

Four basic approaches are recommended by FHWA for collecting travel times for measuring travel time reliability (*6*), including the method of estimating from intelligent transportation systems (ITS) sensor data. The instantaneous model is used to estimate travel times because it is easy to implement and is widely used in practice. Two consecutive ITS sensors directionally act as upstream and downstream end points of a link. For a specific link, designated link i, the values of vehicle speed, collected at time t from upstream and downstream ITS sensors, are denoted $v(i_a, t)$ and $v(i_b, t)$, respectively. The length of link i (l_i) can be measured on the basis of mileage information, and the travel time of this link then can be calculated with Equation 1:

$$\mathrm{TT}(i,t) = \frac{2l_i}{v(i_a,t) + v(i_b,t)} \tag{1}$$

where

l_i = length of link i,
$v(i_a, t)$ = measured speed upstream of link i (i_a) at time t,
$v(i_b, t)$ = measured speed downstream of link i (i_b) at time t, and
$\text{TT}(i, t)$ = estimated travel time of link i at time t.

To reduce the impact of short-duration travel time fluctuations, travel time is aggregated at 5-min intervals. In addition, day of week (DOW) and time of day (TOD) are included in the function of travel time reliability. The matrix in Equation 2 shows the estimated travel times for a link, given specific DOW and TOD:

$$\text{link travel time}(i)\big|_{\text{DOW,TOD}} = \begin{bmatrix} \text{TT}_{1,1} & \text{TT}_{1,2} & \cdots & \text{TT}_{1,m} \\ \text{TT}_{2,1} & \text{TT}_{2,2} & \cdots & \text{TT}_{2,m} \\ \vdots & \vdots & \ddots & \vdots \\ \text{TT}_{n,1} & \text{TT}_{n,2} & \cdots & \text{TT}_{n,m} \end{bmatrix} \quad (2)$$

where link travel time $(i)|_{\text{DOW,TOD}}$ is the estimated travel time of link i at specified DOW and TOD and $\text{TT}_{n,m}$ is the estimated travel time at mth 5-min periods and in nth days. In this study, m equals 12, and n means the number of specific weekdays. Then the number of samples for estimating travel time distribution equals $m * n$.

Because a corridor is composed of several consecutive links, the corridor travel time can be computed as the summation of individual link travel times.

Estimation of Travel Time Distribution

Travel time distribution is considered the starting point of a travel time reliability calculation. Density estimation and smoothing techniques can be used to generate highly adaptable nonparametric probability density functions (PDF). KDE is a well-known approach for estimating the PDF. Rather than fitting predefined parametric analytical statistical distributions, such as Weibull, exponential, lognormal, or Normal distributions, this study uses KDE, a nonparametric statistical model, to estimate the PDF. Equation 3 represents the corresponding kernel estimator. Instead of relying on a few parameters to characterize the entire probability distribution, KDE parameterizes only a localized portion (kernel) of the distribution; it then aggregates the kernels to characterize the population distribution. This approach allows for widely varying manifestations of the distributions to better represent the diversity and fidelity of data.

$$f_h(x) = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right) \quad (3)$$

where

n = number of samples in the data set,
h = smoothing parameter (kernel bandwidth),
$K(*)$ = user-defined kernel function, and
$f_h(x)$ = KDE.

The kernel function, which satisfies the two requirements in Equations 4 and 5, is a nonnegative, real-valued integral function. This study applies the most commonly used approach, employing a Gaussian kernel (versus triangular, etc.) as the seed function to estimate the PDF from the data point travel times. Equation 6 provides the definition of the Gaussian kernel function. Because of aggregation of many localized kernels, the use of a Gaussian kernel does not imply generation of a Gaussian (normal) distribution, or even a symmetric one.

$$\int_{-\infty}^{+\infty} K(u)\,du = 1 \quad (4)$$

$$K(-u) = K(u) \quad (5)$$

$$K(u) = \frac{1}{\sqrt{2\pi}} e^{-1/2u^2} \quad (6)$$

The form of the kernel function is not the sole component of the estimator's performance. Instead, bandwidth or the smoothing factor (h) is the primary parameter of the kernel estimator. The smoothing parameter controls the degree of distribution fidelity retained from the finite number of data points. Larger smoothing factors can be advantageous for filling in portions of the distribution that would otherwise disappear as the number of data points approaches a population. A small change in h can therefore result in a dramatic variation of the estimator. Several methods based either on minimizing asymptotic mean integrated squared error or on cross validation, have been used to determine the smoothing factor. Equation 7 shows one optimal solution (*23*):

$$h = \left(\frac{4\delta^5}{3n}\right)^{1/5} \quad (7)$$

where δ is the standard deviation of the data points of travel times.

Since the data points for travel times on a selected link or a corridor can be computed with Equation 2, the travel time distribution can be readily determined with the KDE technique. The time frame in measuring travel time reliability is based on DOW and TOD. Although not included in this study, use of Equation 2 with KDE allows for identification of preferred smoothing parameters by comparison of distributions generated with larger and smaller numbers of data points.

Reliability Index Calculation

The HL-RF algorithm is widely used in the field of reliability engineering (*24*). It computes the performance reliability index of a system described by a function of statistically independent random variables. The system performance is expressed by a limit-state function, $g(X)$, where $X = (X_1, X_2, \ldots, X_n)$, and each X_i represents a normally distributed, independent random variable. A set of realized values for X, denoted as $x = (x_1, x_2, \ldots, x_n)$, is used to examine whether the value of the limit-state function is positive or negative. The three resulting states are classified as follows: (*a*) when $g(x) > 0$, it is defined as a safe state; (*b*) when $g(x) < 0$, it is defined as a failure state; and (*c*) when $g(x) = 0$, it is defined as the limit state, which is graphically visualized as a failure surface in X space. The failure surface can be built by solving for $g(X) = 0$. The reliability index, β, of the system represents the minimal distance from the origin, in normalized space, to a point on the failure surface. Since the limit-state function $g(X_1, X_2, \ldots, X_n)$ may be nonlinear, an iterative procedure is designed for finding convergence in β, and any standard optimization algorithm (e.g., nonlinear programming) can be used to obtain its value by minimizing β subject to the constraint $g(X) = 0$. The reliability index β can be interpreted as the number of standard deviations from the mean performance value to the closest point on the failure surface (when the random variables are normalized to have zero mean and unit standard deviation). It is also the variate of the standard normal cumulative distribution function (CDF) at the most probable point of failure for $g(X)$, which implies that the reliability is $\Phi(\beta)$, where Φ is the CDF of the standard normal distribution.

One complication of the preceding is that each X_i in the limit-state function is intended to be normally distributed. Many variables in reality, however, do not follow a normal distribution (including any parameter that is strictly positive). A transformation of nonnormal variables to equivalent normal variables is therefore added as the distinguishing component in the iterative procedure of HL-RF versus the HL method. The following section describes the application of the HL-RF algorithm to assessment of travel time reliability.

HL-RF Algorithm

Assume a system consists of n components, say, $X_1, X_2, \ldots, X_n$, each component being associated with a failure distribution. Each X_i should follow a normal distribution and be independent of the other. Assuming the system performance can be expressed as $g(X)$, where $X = (X_1, X_2, \ldots, X_n)$, the resulting mean value and standard deviation of X_i are denoted as μ_{X_i} and σ_{X_i}.

For computing a converged reliability index, β, the iterative procedure starts with a design point $X^* = [X_1^*, X_2^*, \ldots, X_n^*]$, where X_i^* are usually initially assigned as the mean values, μ_{X_i}. The transformation from nonnormal to normal variables is executed at each iterative design point. For example, consider μ_{X_i} and σ_{X_i} as the mean and standard deviation of a nonnormal variable X_i. At iterative points, nonnormal variable X_i is transformed into an equivalent normal variable, denoted X_i^N, having mean and standard deviation denoted as $\mu_{X_i}^N$ and $\sigma_{X_i}^N$, respectively. Reduced variables ($Z^* = [Z_1^*, Z_2^*, \ldots, Z_n^*]$ then can be projected from normal variables ($X^* = [X_1^*, X_2^*, \ldots, X_n^*]$ with Equation 8. The reliability index, β, is computed at each iteration with Equation 9.

$$Z_i^* = \frac{X_i^* - \mu_{X_i}^N}{\sigma_{X_i}^N} \tag{8}$$

$$\beta = \frac{G^T Z^*}{\sqrt{G^T G}} \tag{9}$$

where

$$[G] = \begin{bmatrix} [G_1] \\ [G_2] \\ \vdots \\ [G_n] \end{bmatrix} \text{ and } G_i = -\frac{\partial g}{\partial Z_i} = -\frac{\partial g}{\partial X_i}\frac{\partial X_i}{\partial Z_i} = -\frac{\partial g}{\partial X_i}\sigma_{x_i}\Bigg|_{\text{evaluated at design point}}$$

The vector G in Equation 9 represents the gradient or sensitivity of the performance function, $g(X)$, with respect to each random variable at the current (iterative) design point and T is the transpose of the vector or matrix. Once each reliability index β is calculated, its convergence determines the termination state of the iteration. If β converges to within a user-defined threshold, the iteration ends; otherwise a new design point is computed on the basis of new reduced variables for the next iteration. Equation 10 shows the procedure for obtaining new reduced variables. Equation 11 is then used to derive a new design point from the new reduced variables.

$$Z^* = \beta\alpha \tag{10}$$

where

β = computed reliability index in the current iteration,
$\alpha = G/\sqrt{(G^T G)}$, and
Z^* = new reduced variables.

$$X_i = Z_i \sigma_{X_i} + \mu_{X_i} \tag{11}$$

The remaining task is to transform nonnormal random variables into equivalent normal random variables at the new design point.

Transformation of Nonnormal Random Variables to Equivalent Normal Random Variables

Rackwitz and Fiessler claimed two requirements for the transformation: (*a*) the CDF of the original and equivalent normal variables should be matched at the required point, and (*b*) the PDF of both variables should also be matched (*24*). Equation 12 establishes the first requirement:

$$F_X(x^*) = \Phi\left(\frac{x^* - \mu_X^N}{\sigma_X^N}\right) \tag{12}$$

where

x^* = value of X^* at the required point,
$F_X(*)$ = CDF of the nonnormal distribution,
μ_X^N = mean value of the equivalent normal distribution,
σ_X^N = standard deviation of the equivalent normal distribution, and
$\Phi(*)$ = CDF of a standard normal distribution.

Rearranging terms in Equation 12 yields two forms shown as Equations 13*a* and 13*b*:

$$\mu_X^N = x^* - \Phi^{-1}\left[F_X(x^*)\right]\sigma_X^N \tag{13a}$$

$$\frac{x^* - \mu_X^N}{\sigma_X^N} = \Phi^{-1}\left[F_X(x^*)\right] \tag{13b}$$

where $\Phi^{-1}(*)$ is the inverse CDF of the standard normal distribution.

The second requirement is expressed by Equation 14:

$$f_X(x^*) = \frac{1}{\sigma_X^N}\phi\left(\frac{x^* - \mu_X^N}{\sigma_X^N}\right) \tag{14}$$

where $f_X(*)$ is the PDF of the nonnormal distribution and $\phi(*)$ is the PDF of a standard normal distribution.

Similarly, Equation 14 can be rewritten as Equation 15:

$$\sigma_X^N = \frac{\phi\left(\dfrac{x^* - \mu_X^N}{\sigma_X^N}\right)}{f_X(x^*)} \tag{15}$$

Substituting Equation 13*b* into Equation 15 produces Equation 16:

$$\sigma_X^N = \frac{\phi\left(\Phi^{-1}\left[F_X(x^*)\right]\right)}{f_X(x^*)} \tag{16}$$

Equation 13*a* can be rearranged by substituting Equation 16 for Equation 17:

$$\mu_X^N = x^* - \Phi^{-1}\left[F_X(x^*)\right]\frac{\phi\left(\Phi^{-1}\left[F_X(x^*)\right]\right)}{f_X(x^*)} \tag{17}$$

Thus, the transformation from nonnormal variables to normal variables at a required point can be expressed with the two equivalent, definitional parameters of a normal distribution, equivalent mean (μ_X^N) and standard deviation (σ_X^N). This transformation is required at each iteration of the HL-RF algorithm, which implies that μ_X^N and σ_X^N may change accordingly.

Corridor Travel Time Reliability

Travel time reliability in this study is in the sense of on-time performance. A value of the reliability may be given according to travelers' anticipated travel time on a corridor at a specific DOW and TOD. Because the selected corridor travel time distribution can be estimated by the KDE technique, X is referred to as the random variable following the estimated corridor travel time distribution. The limit-state function in this single corridor study is therefore defined simply as

$$g(X) = \text{ATT} - X \tag{18}$$

where ATT is a traveler's anticipated travel time on a selected corridor (a constant).

Because Equation 18 is a linear function, the reliability index, β, may converge in one iteration, unless the equivalent normal means and standard deviations of the original distributions change significantly after the first iteration. Combining Equation 18 with the computing procedure for reliability index calculation, β is equal to ATT $- \mu_X^N/\sigma^{2N}_X$; the travel time reliability then can be calculated as $\Phi(\beta)$, where $\Phi(*)$ is the CDF of the standard normal distribution. The travel time distributions, however, are characterized by a specific time period, that is, DOW and TOD. With this formulation, the input parameters are included to obtain Equation 19:

$$\text{travel time reliability }(\text{ATT})\big|_{\text{DOW,TOD}} = \Phi(\beta) = \Phi\left(\frac{\text{ATT} - \mu_X^N}{\sigma_X^N}\right) \tag{19}$$

Equation 19 indicates that the travel time reliability of a particular single corridor, given a time period, can be represented as a value of the CDF of the standard normal distribution. In terms of the selection of the initial design point, any point falling into the range of the travel times works well; this study uses the mean of travel time as the initial design point. For this simple case of a single corridor and specific time period having a single distribution, $\Phi(\beta)$ represents a statement equivalent to $F_X(\text{ATT})$, because X is a single random variable in this case. The primary benefit of the HL-RF method becomes evident when X is a vector of random variables representing distributions for multiple corridors.

Network Travel Time Reliability

Travelers often change freeways during a route before reaching their destinations. Network travel time reliability is essential for long trips that use multiple freeways. The network travel time reliability can be computed based on the multiple individual corridors on different freeways. An underlying assumption in this case is that the traffic flow relationships of corridors on various freeways are independent. Although the independence assumption exists with regard to the random variables, that separate travel time distributions are generated on the basis of real-life traffic data at specific TOD and DOW mitigates the effect of this assumption. Indeed, as illustrated later, there is a clear correlation between the travel time distributions for the same TOD and DOW on different corridors. Therefore, the assumption of independent random variables may be justifiable. From this assumption, the network travel time reliability is considered as an extension of the corridor travel time reliability problem. The limit-state function, which is built on travel time distributions of m corridors, is defined as follows:

$$g(X_1, X_2, \ldots, X_m) = \text{ATT} - X_1 - X_2 \cdots - X_m \tag{20}$$

where X_i is the random variables depicting the ith corridor estimated travel time distribution.

Equation 21 shows the network travel time reliability given a specific DOW–TOD time period and based on the limit-state function shown in Equation 20:

$$\text{travel time reliability }(\text{ATT})\big|_{\text{DOW,TOD}} = \Phi(\beta) = \Phi\left(\frac{\text{ATT} - \sum_{i=1}^{m}\mu_{X_i}^N}{\sqrt{\sum_{i=1}^{m}\sigma^{2N}_{X_i}}}\right) \tag{21}$$

where m is the number of corridors on different freeways.

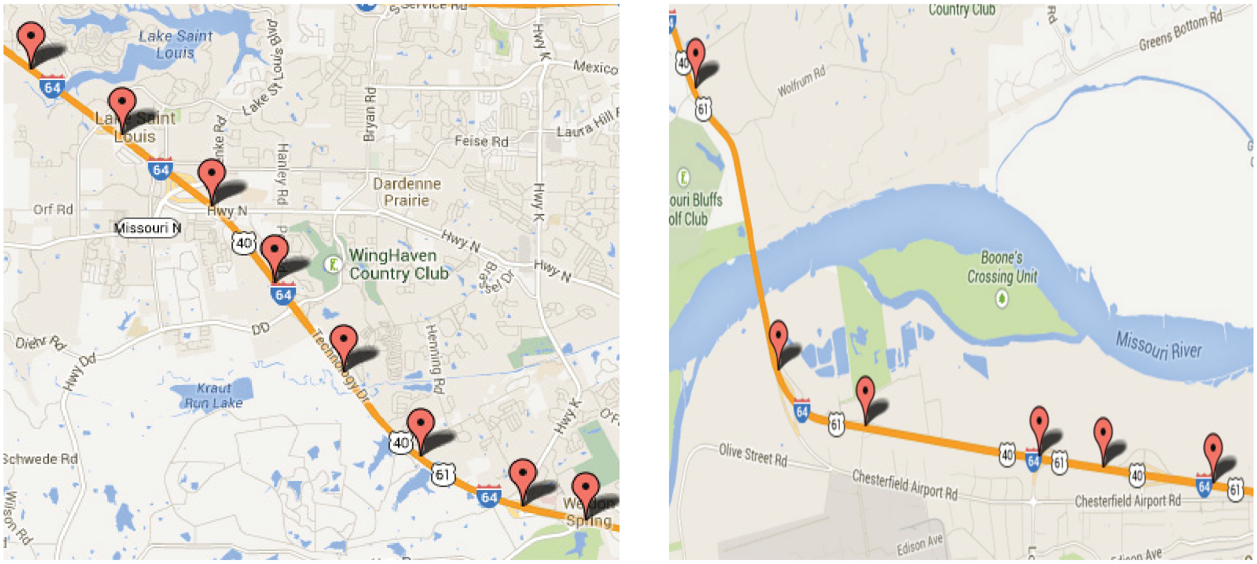
The proposed corridor travel time reliability model is a special case of the proposed network corridor travel time reliability model.

IMPLEMENTATION AND APPLICATIONS

Study Data

The Missouri Department of Transportation is in charge of deployment and maintenance of more than 700 traffic sensors on freeways in the greater Saint Louis area. The department's Transportation Management Center monitors traffic flow conditions by collecting vehicle speed, traffic volume, and occupancy information.

Two specific corridors on westbound I-64 in Saint Louis are used to build travel time distributions and calculate corresponding travel time reliabilities. The locations of the two corridors, designated as Corridor 1 and Corridor 2, are illustrated in Figure 1. The total lengths



(a) (b)

FIGURE 1 I-64 study corridors: (*a*) Corridor 1, westbound from Highway K to Prospect Road, and (*b*) Corridor 2, westbound from Chesterfield Parkway to Research Park Drive. (Source: background image, https://maps.google.com.)

of the two study corridors are 8.02 mi and 6.80 mi, respectively. It is generally accepted that peak hours occur in the late afternoon. When the vehicle volume exceeds the designed capacity of roadways, or the designed capacity is degraded because of some cause such as incidents, traffic congestion may occur. Travel time distribution and relevant travel time reliability may vary greatly under different traffic conditions. Corridor 1 sometimes experiences nonrecurring traffic congestion in the afternoon peak hours, and Corridor 2 suffers heavy traffic congestion in afternoon peak hours on weekdays. A potential cause for the recurring traffic congestion on Corridor 2 is the narrow lanes of the Daniel Boone Bridge over the Missouri River. The weekday traffic data from June 1, 2012, to April 1, 2013, were selected as the study data set. Therefore, the parameter n in Equation 2 is 40.

Travel Time Distribution

A custom MATLAB program was built for computing the travel times on the two selected corridors for each DOW and TOD. The travel time distributions were estimated with the KDE technique with optimized bandwidths. Figures 2 and 3 illustrate the travel time distributions of Corridor 1 and Corridor 2 every weekday for when TOD is 9 a.m. and when TOD is 5 p.m., respectively. Traffic free-flow and congested conditions are distinctly shown, respectively, during the two time periods. Many distributions in the figures are clearly right skewed (for example, Figure 2, *f*, *g*, and *h*); however, others appear to be neither left nor right skewed significantly. The results confirm that no unique common theoretical probability distribution could be used to represent the travel time distributions.

Furthermore, compared with the distributions during the two time periods, it was found that (*a*) the travel speed may vary largely depending on the individual traveler in a traffic free-flow condition—travelers had limited choices on the drive speed when traffic congestion occurred, leading to synchronized speed on corridors, and (*b*) a small collection of relatively large travel times (they are not outliers) at the afternoon peak affected construction of the corresponding travel time distributions. These relatively large travel time values may have been recorded during incidents, such as vehicle crashes and heavy snow.

Corridor Travel Time Reliability

Figure 4 shows the travel time reliability calculated for a given time period for the two selected corridors when an anticipated travel time of the corridor is factored in. Figure 4, *a* and *b*, presents the travel time reliability distribution of each weekday on Corridor 1 during the two time periods. Given a specific anticipated travel time, the five values of the travel time reliability show less variance in Figure 4*a* than in Figure 4*b*. For example, with a traveler's anticipated travel time set as 8 min 30 s when TOD is 9 a.m., the resulting reliability values on each weekday are 92% (Monday), 94% (Tuesday), 95% (Wednesday), 96% (Thursday), and 95% (Friday). These five reliability values are not distinguishable because the corresponding travel time distributions shown in Figure 2, *a* through *e*, are concentrated around data points with a same travel time value, and the distributions are bounded within a small range.

However, when the anticipated travel time is 17 min in the afternoon peak hour, the weekday reliability values are computed as 96% (Monday), 98% (Tuesday), 45% (Wednesday), 99% (Thursday), and 34% (Friday). In Figure 3, *a* through *e,* the mode of each single distribution is around 12 min. However, the ranges of distributions on Wednesdays (8 ~ 35 min) and Fridays (8 ~ 42 min) are wider than the others (roughly, 8 ~ 20 min), indicating the two distributions have much more variance, and the traffic flow may be unstable during peak hours on Wednesdays and Fridays. Therefore, the reliability on Wednesdays and Fridays is lower than that on other weekdays in the afternoon peak hour.

The standard deviation distributions of the travel time reliability for each weekday, based on the anticipated travel time, are also shown in Figure 4, *a* and *b*. Scaled flatter standard deviation distribution indicates the traffic flow condition performs more similarly on

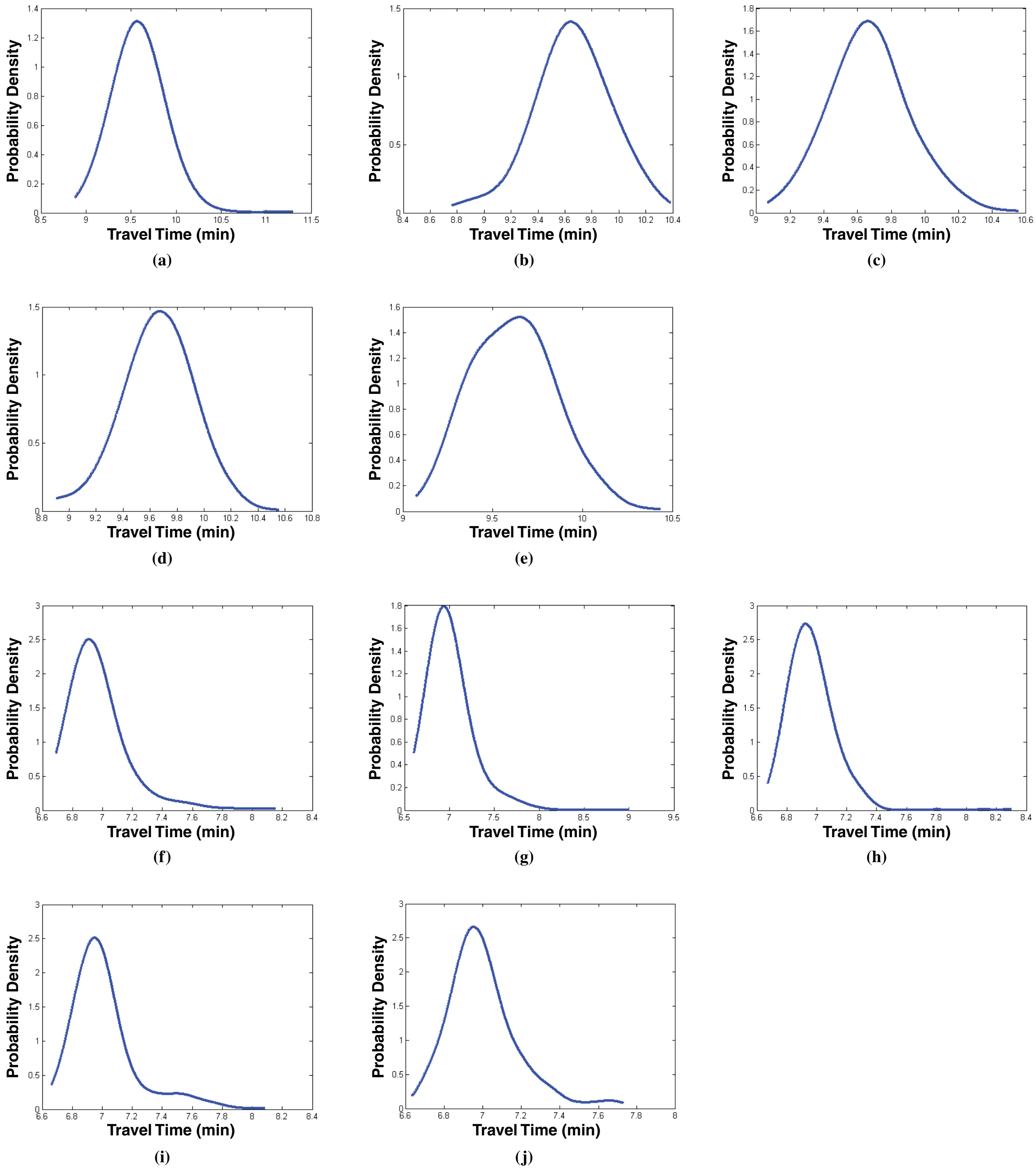


FIGURE 2 Travel time distributions estimated by KDE for TOD of 9 a.m. on Corridor 1: (*a*) Monday, (*b*) Tuesday, (*c*) Wednesday, (*d*) Thursday, and (*e*) Friday; Corridor 2: (*f*) Monday, (*g*) Tuesday, (*h*) Wednesday, (*i*) Thursday, and (*j*) Friday.

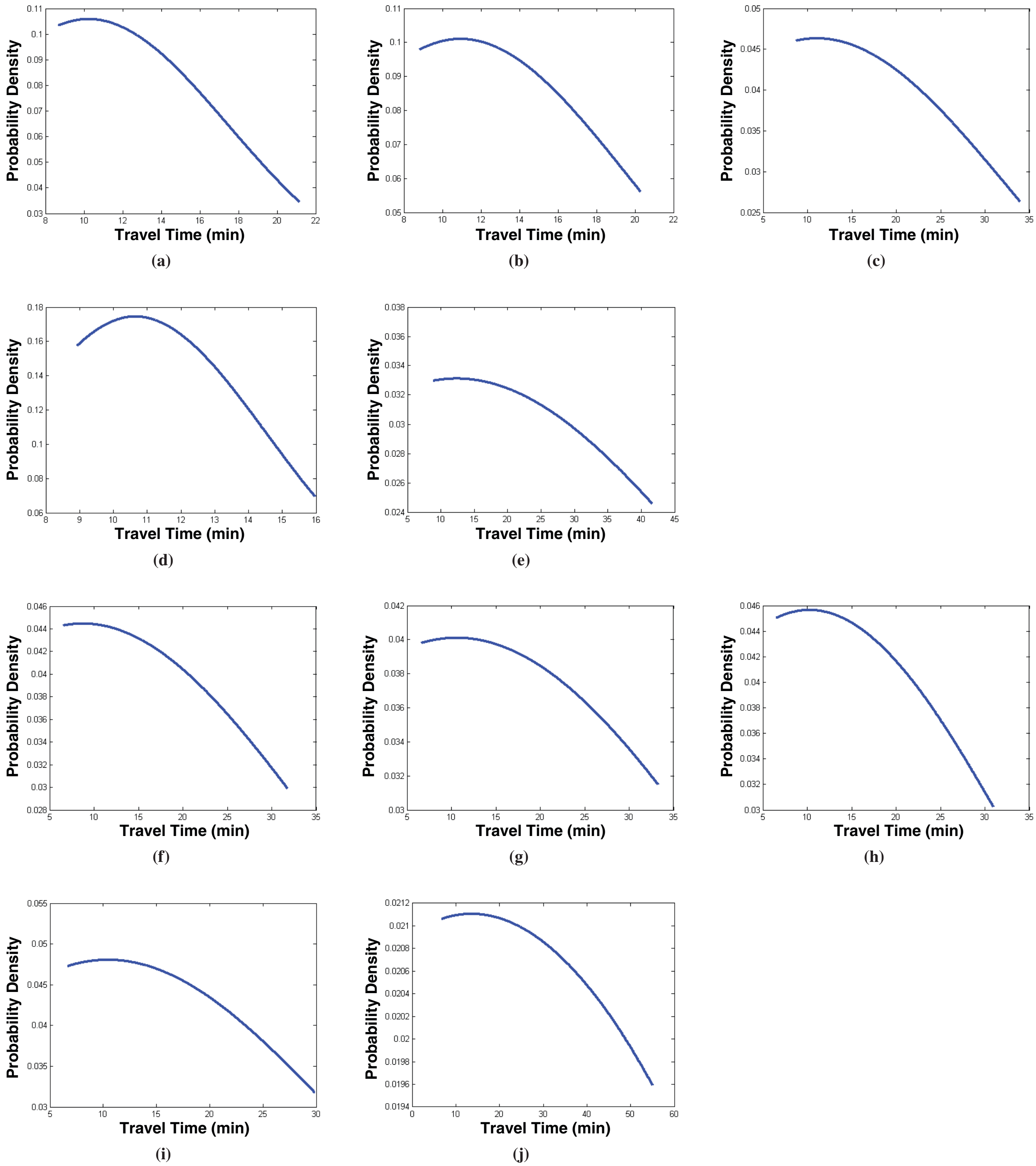


FIGURE 3 Travel time distributions estimated by KDE for TOD of 5 p.m. on Corridor 1: (*a*) Monday, (*b*) Tuesday, (*c*) Wednesday, (*d*) Thursday, and (*e*) Friday; Corridor 2: (*f*) Monday, (*g*) Tuesday, (*h*) Wednesday, (*i*) Thursday, and (*j*) Friday.

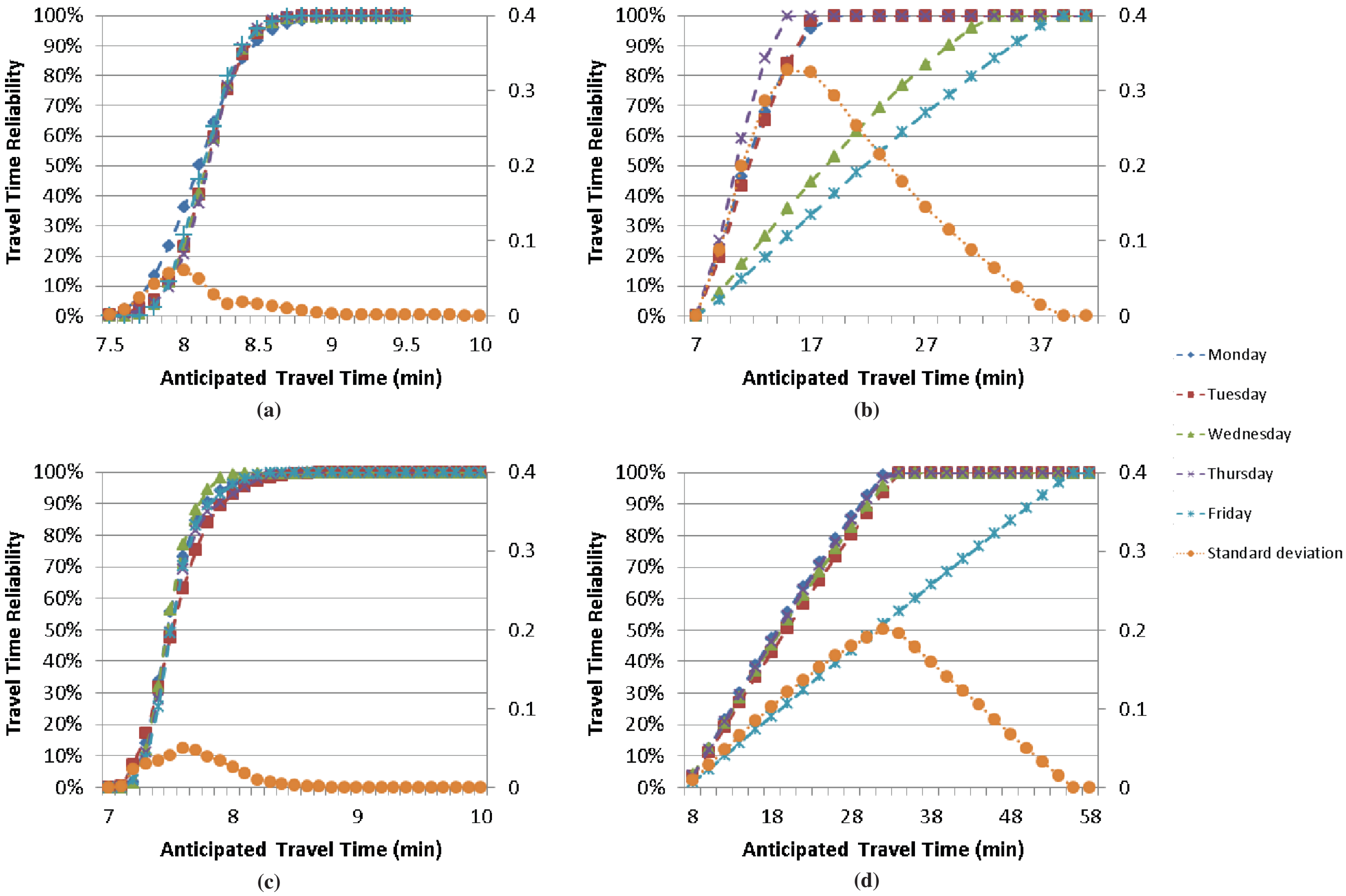


FIGURE 4 Travel time reliability at given anticipated travel time: Corridor 1 (*top*), (*a*) TOD = 9 a.m. and (*b*) TOD = 5 p.m., and Corridor 2 (*bottom*), (*c*) TOD = 9 a.m. and (*d*) TOD = 5 p.m.

weekdays at the same time period, which may allow travelers to feel that traffic conditions are more stable. Therefore, throughout the week, there is significantly greater deviation in the travel time reliability in the afternoon peak hour than in the morning hour on Corridor 1. Similarly, Figure 4, *c* and *d*, presents travel time reliability distributions on Corridor 2. The corresponding standard deviation distributions also indicate smaller variations in travel time reliability between 9 and 10 a.m., and greater variations occur in the afternoon peak hour.

The proposed travel time reliability method emphasizes travelers' perspectives (anticipation), and it can help travelers perceive how reliable trips can be made, on time, for selected corridors within a specific time period. For example, the results of the proposed reliability method could inform a traveler who intends to travel on Corridor 1 that he or she would spend 9 min in the morning with relatively high reliability (probability); however, 9 min may not be enough to travel through Corridor 1 during the afternoon peak hour because of the lower reliability.

Comparisons of Travel Time Reliability Estimation Methods

Because the Florida reliability method also was developed from the traveler's perspective (*25*), the results of the Florida method are compared with the results of the proposed travel time reliability method. It is uncertain, however, which percentage above the median travel time should be used, and therefore several additional percentages (Δ) over median travel time—5%, 10%, 15%, and 20%—are considered as anticipated travel time. The same data set is used for all 17 TODs and 5 DOWs, and each method is executed to compute the percentage of reliable travel on the two corridors. Figure 5 shows a comparison of the results generated by the two methods on the two corridors on Wednesday.

Two features can be identified in Figure 5. First, as the anticipated travel time is increased, the travel time reliability generated from both methods increases. Second, the travel time reliability generated from the proposed method is found to be lower than that generated from the Florida reliability method, especially during peak hours. This is because the Florida reliability method is based on the probability mass function, where the travel time is considered as discrete random variables. Probability mass functions cannot fully represent travel time distributions, and critical high-fidelity information, such as steep localized distribution gradients, may be lost, especially when the travel time data set is not huge. However, in reality, travel time should be modeled with continuous random variables. The proposed method uses the continuous feature of travel time distributions generated from KDE. Therefore, the proposed method can better capture the detailed variability of traffic flow, especially the instability during peak hours.

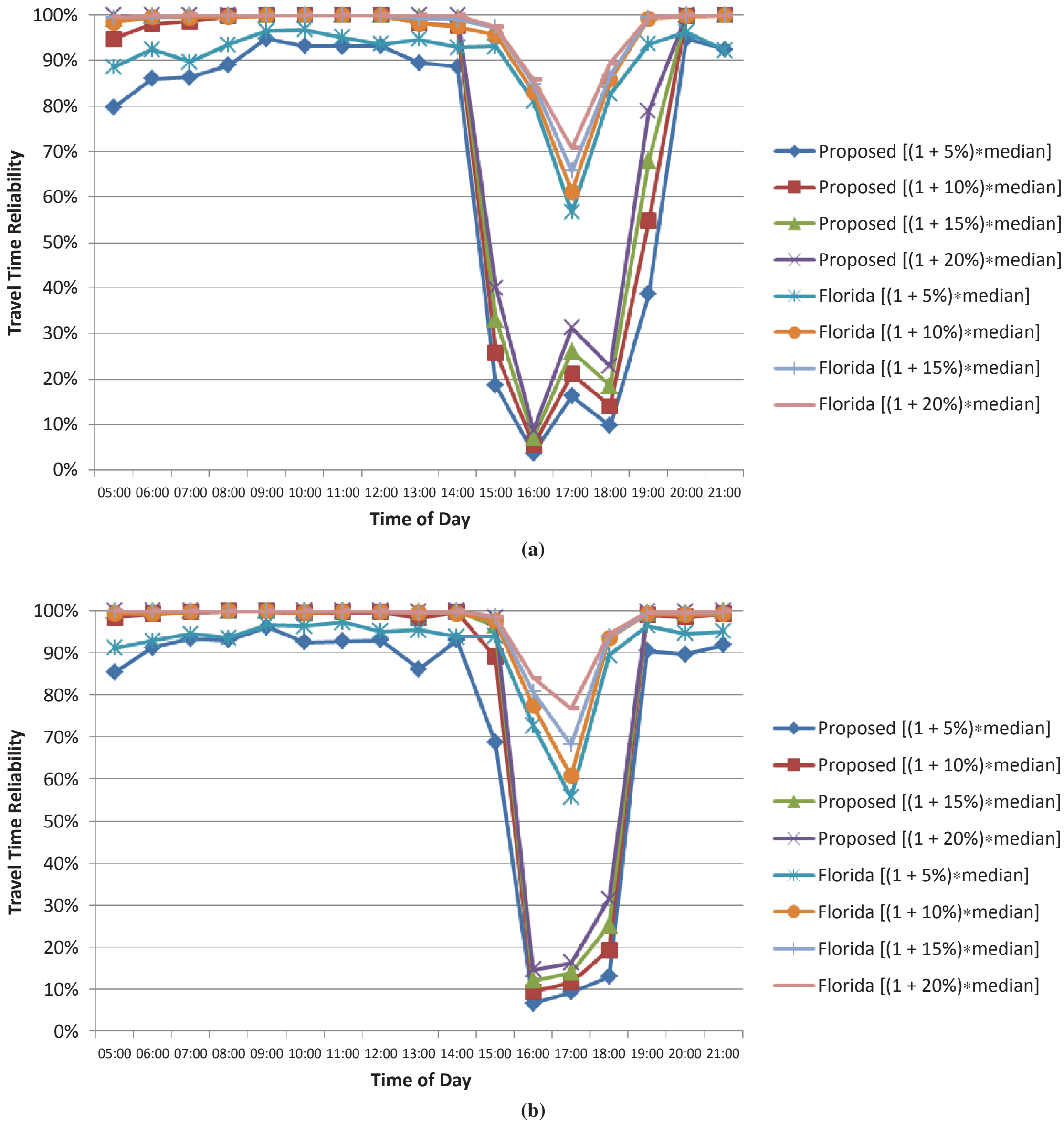


FIGURE 5 Results comparison for selected Wednesdays: (*a*) Corridor 1 and (*b*) Corridor 2.

CONCLUSIONS

Because of the large availability of real-life traffic data on freeways in Saint Louis, new methodologies were developed that make use of traffic data instead of simulation data. The nonparametric technique of KDE was used to estimate the travel time distribution given specific DOW and TOD periods, rather than conventional predefined parametric probability distributions such as single model (e.g., the lognormal distribution) or mixture model (e.g., two-state Normal distribution), to estimate travel time distribution. The KDE technique may offer greater flexibility in travel time distributions and may be more suitable given the availability of real-time traffic data. It eliminates the challenging requirement to identify and fit traffic data into predefined theoretical probability distributions. Also, KDE can be used for a wide variety of situations, especially when the traffic flow conditions are unknown ahead of time. The process of estimating travel time distribution with KDE for varied traffic flow conditions was considered as a generalization of the estimation process. Finally, KDE is more adaptable for constructing the travel time distribution of a user-defined route. The use of KDE could be limited when the number of data points is relatively small or outliers exist, because changes in the smoothing factor become more influential in those cases. Data preprocessing could be implemented to address this.

The HL-RF algorithm was used to calculate travel time reliability. First, the computing procedure for travel time reliability of corridors on a freeway was introduced. Network travel time reliability was then developed and was viewed as a generalized version of the corridor travel time reliability. The resulting equations for both corridor and network travel time reliability were presented as a value of the CDF for standard Normal distributions at the point of anticipated travel time. The latter required a transformation into equivalent standard

Normal variable form. Given probability distributions and an anticipated travel time from travelers, the two equations of the corridor and network travel time reliability can be used to address the question, how reliable is my proposed trip time? The definition of travel time reliability is in the sense of on-time performance, and this study was conducted from the perspective of travelers.

In comparison with the results of the Florida reliability method with four thresholds, the results showed that the new method performed similarly during nonpeak hours; however, it could capture detailed variability during peak hours, resulting in a major distinction between the Florida reliability method and the proposed method. The major advantages of the proposed method are that it (*a*) demonstrated an alternative way to estimate travel time distributions when the choice of probability distribution family is still uncertain and (*b*) showed its flexibility for application to different levels of roadways (e.g., individual roadway segment or network).

Further research will use more real-life data to analyze the sensitivity of this method and will attempt to address the following questions:

- How large a data set and what requirements are needed to estimate the travel time distribution and calculate the reliability accurately?
- Which method for smoothing factor identification can best meet the objectives and needs of this research?
- What are the specific differences when the KDE method is applied, versus predefined parametric distributions, on the same data set?

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