

Research Article



# Examining Left-Turn Speeds for Intersection Signal Timing Design Using Crowdsourced Trajectory Data

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#### **Abstract**

Left-turn vehicles' intersection entry speeds are critical in determining yellow and red clearance intervals. The signal timing design guidelines recommend collecting the left-turn entry speed or estimating it based on the speed limit. Estimations based on speed limits have generally been effective but can result in over- or underestimations at intersections with varying geometric characteristics. It is critical for agencies to identify geometric characteristics that could influence left-turning vehicles' entry speeds. This awareness could inform decision making by pinpointing intersections requiring field data collection to estimate yellow and clearance intervals instead of estimation based on the speed limit. This paper aims to evaluate geometric characteristics influencing left-turning vehicle speeds. This study examined left-turning speeds at 60 signalized intersections in Tucson, Arizona. Crowdsourced trajectory data offer a cost-effective and scalable approach to capturing real-world driving behavior, making them particularly valuable for large-scale safety and operations studies. Ordinary least squares (OLS) and quantile regression models were used to analyze speed profiles and the influence of various factors. Findings revealed that the number of left-turn lanes, medians, and the width of the left-turn lane affect entry, mid-maneuver, and exit speeds for movements impeded by traffic queues at intersections. These results emphasize the importance of considering intersection geometry and speed limit during signal timing design. Future research could explore the impact of other vehicles, signal timing phases, demographic characteristics, and vehicle classifications to deepen our understanding of driver behavior in left-turn movements, thereby improving intersection design and traffic management strategies.

## **Keywords**

operations, ITS, arterial, signal phase, signalized intersection, traffic signal

The computation of traffic signal intervals, particularly yellow and red clearance intervals, has long been a complex issue for practitioners because of factors such as driver comfort, perception, and regulatory interpretation. While various publications offer guidance, there is no nationally recognized standard for determining the length of change intervals. Key resources, such as the Institute of Transportation Engineers (ITE) Traffic Engineering Handbook and the Manual on Uniform Traffic Control Devices (MUTCD) provide valuable recommendations. The ITE suggests using kinematic equations to calculate yellow and red clearance intervals. Determination of yellow and red clearance intervals based on the kinematic equation requires several parameters, including perception-reaction time, the 85th

percentile of approach speed, and approach grade. While some of these parameters could be considered globally, other critical parameters, such as the intersection left-turn entry speed, require local values. The ITE 2020 Guidelines and NCHRP Report 731 recommend collecting the left-turn entry speed data or estimating it based on the speed limit (1, 2). The NCHRP Report suggests that the 85th percentile speed for left-turning vehicles is

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estimated as the approach speed limit minus 5 mph (1). The ITE 2020 guidelines recommend using the speed limit as the 85th percentile approach speed for left-turn movements and 20 mph as the intersection entry speed for left-turn movements (2-4). In 2022, Jerome et al. assessed the extended kinematic equation proposed by the ITE 2020 guidelines for estimating yellow intervals for left-turn movements. Their analysis, based on 262 real-world trajectories of free-flowing left-turning vehicles, suggested that the equation tends to overestimate the required yellow interval. This overestimation occurs because the critical distance is effectively reduced when left-turning vehicles begin decelerating before reaching the braking point, and the average traversing speed remains higher as a result of vehicles decelerating at moderate rates rather than the maximum rate assumed in the equation. Despite these findings, the study faced limitations, including its exclusive focus on free-flowing vehicles, which does not account for impeded traffic conditions. Additionally, the results were derived from a limited number of study sites, raising concerns about their applicability to other intersection types (5).

Estimations based on speed limits have generally been effective but can result in over- or underestimations at intersections with varying geometric characteristics, such as dual left-turn lanes, longer turning radii, and wider intersections. There is evidence that factors such as intersection geometry, drivers' instinctive judgments, and their targeted exit lanes cause significant variations in the paths and speeds of left-turning vehicles (6, 7). For example, while Dias et al. (2020) found that minimum speeds were similar across various curve radii, the standard deviations of speed increased with larger intersection angles (6). It is critical for agencies to identify geometric characteristics that could influence the left-turning vehicles' entry speeds. This awareness could inform decision making by pinpointing intersections requiring field data collection to estimate clearance intervals instead of estimation based on the speed limit.

Collecting field speed data helps avoid the estimation of the entry speed based on kinematic equations and the speed limit. However, the challenges for most agencies in collecting speed data are the cost, time, and scalability. It is difficult for most agencies to collect data for several intersections in the network to determine the yellow interval using radar guns or video-based sensors. For example, using radar guns, Yu et al. (2004) collected data on only 19 intersections with speed limits ranging from 40 to 55 mph and 125 vehicle trajectories (8). The advances in the availability of crowdsourced trajectory data could reduce the cost and simplify the scalability of the yellow and red clearance interval estimation. Moreover, the availability of a considerable data set could help develop models that could be used to predict

the traffic entrance speeds based on geometric characteristics and applied in areas that lack the means and funds to acquire the crowdsourced data.

This paper uses crowdsourced trajectory data to evaluate geometric characteristics at signalized intersections influencing left-turning vehicle speeds. Also, the study demonstrates the applicability of the crowdsourced trajectory data in determining the left-turn entry speed. While the main focus is on entry speed, this research aims to assist agencies in understanding and estimating the speed profile of left-turning movements by including the analysis of speeds during the middle of the left-turning maneuver and when exiting the intersection after executing the left-turn maneuver. The left-turning vehicles' speed information at signalized intersections could be used in other areas, including realistic representation of surrounding vehicle movements in driving simulators, virtual reality applications, and microscopic simulation tools.

## Literature Review

Previous research has extensively explored the trajectories of turning movements. Abdeljaber et al. (2020) manually extracted 44 trajectories of free-flowing vehicles from recorded video at a signalized intersection in Qatar. Results indicated that the characteristics of these paths were significantly influenced by a vehicle's entry speed, minimum speed during the turn, and the lateral distance between the exit point and the curb. Approximately 70% of drivers preferred the middle lane when exiting the intersection. This study suggested that the speed parameters can indicate driver aggressiveness, and that both driver behavior and the chosen exit lane affect the turning vehicle's trajectory (9). Dias et al. (2020) investigated left- and right-turn trajectories under free-flow conditions in left-hand traffic. It was found that while the curve radius does not affect average speed and variation, accelerations are sensitive to it. Additionally, vehicle paths become less varied as the intersection angle increases, the trajectories shift inward as the exit speed decreases, and vehicle paths move toward the inner corner of the turn with decreasing entry acceleration (6).

Fitzpatrick et al. (2021) video recorded the speeds of 4,394 right-turning vehicles at 31 urban signalized intersection approaches in Texas. A log-normal model was developed to predict right-turn speeds based on various site characteristics, including curb radius, leading headway, vehicle type (car versus truck), maneuver of the preceding vehicle (through versus right turn), and signal indication (yellow or green). The analysis provided strong evidence that right-turning speed is influenced by the corner radius: larger radii correspond to higher turning speeds. Also, right-turn speeds tend to increase

slightly when the preceding vehicle goes straight through the intersection, or when the signal indication is yellow rather than green. The study revealed that the calculated 85th percentile turning speed is generally higher than the assumed speed derived from the radius of curvature equation, and confirmed that trucks turn more slowly than cars (10).

Dolatalizadeh et al. (2020) examined driver behavior during left-turning movements at unsignalized intersections. Using data from 353 left-turning vehicles collected via a fixed digital camera, vehicle speed profiles were categorized into three types based on their descending and ascending patterns. Key findings include the influence of initial speed and exposure on the speed profile. Vehicles with left exposure at intersections showed a tendency for an initial descending slope followed by an ascending slope in the speed profile. For vehicles with high initial speeds, drivers typically maintained a smoother speed profile with slight descending and ascending trends. Conversely, vehicles with low initial speeds and fewer exposures often had consistently ascending speed profiles (11). Wolfermann et al. evaluated the speed profiles of right- and left-turning vehicles at signalized intersections by manually tracking vehicle positions and times from video recordings, analyzed using the TrafficAnalyzer image processing program. The study tracked 117 vehicles across 18 approaches and used regression models to analyze speed, acceleration, and jerk profiles. Results demonstrated that speed profiles are sensitive to intersection layout, including the approach angle, curb radius, and position of the hard nose (7).

Laureshyn et al. (2009) classified the speed profiles of left-turning vehicles at a signalized intersection using pattern recognition techniques such as cluster analysis, supervised learning, and dimension reduction. Three traffic scenarios were identified: no oncoming traffic, yielding to oncoming vehicles, and yielding to pedestrians. The findings confirmed the efficacy of pattern recognition techniques in classifying speed profiles (12). Alhajyaseen et al. (2013) studied the trajectory distribution of turning vehicles as a function of intersection geometry, vehicle type, and speed. Alhajyaseen et al. (2013) found that the paths of right-turning vehicles in left-hand traffic were more sensitive to vehicle speed and turning angle, whereas left-turning vehicle paths were more influenced by the intersection corner radius, turning angle, and vehicle speed (13).

While several studies investigated turning vehicles' trajectory and speed profiles, most have focused on individual vehicle paths and recognition, often within left-hand traffic scenarios. Furthermore, these studies frequently examined turning vehicles under free-flow conditions, without interactions with other road users.

There is also a notable lack of research conducted in the United States, where signalized intersections have unique operational characteristics. This gap underscores the need for further research on typical U.S. signalized intersections. This study addresses this gap by employing a quantile regression modeling approach on crowdsourced trajectory data from 32,884 left-turning vehicles at 60 signalized intersections in Tucson, Arizona. The findings will provide transportation agencies with a deeper understanding of the factors influencing the speed profiles of left-turning vehicles, aiding in establishing parameters needed for effective signal timing design.

The paper is organized as follows: the next section outlines the selection of study sites. This is followed by a detailed description of the data collection and processing procedures, including the identification of left-turning vehicles, speed data collection, and the geometric characteristics of intersections. The subsequent section presents the modeling approach and analysis of the results. Finally, the study concludes by discussing the findings and their broader implications. Figure 1 provides a flow chart illustrating this study's overall structure and process.

# **Study Sites**

This study sought to identify geometric characteristics influencing left-turning vehicle speeds and how the entrance speeds vary at different positions at the intersection during the left-turning maneuver. Signalized intersections in Tucson, Arizona served as the focus of this analysis. A diverse set of intersections was selected to ensure a comprehensive analysis incorporating a wide range of geometric characteristics. Since the City of Tucson manages more than 450 signalized intersections, crash experience was used as the primary selection criterion. This method allowed for the inclusion of various intersection types while mitigating potential biases that could arise from selecting sites based solely on geometric attributes. For this study, 60 standard four-way signalized intersections in Tucson, Arizona were selected from a pool of more than 450 intersections after ranking them based on crash frequency and severity. The top 20 intersections in the ranking (i.e., the most unsafe signalized intersections based on crash frequency and severity), the lowest 20 intersections (i.e., the safest signalized intersections based on crash frequency and severity), and the middle 20 intersections were selected, totaling 60 intersections. This selection of study sites provided a balanced representation of intersections with different levels of crash risk. Figure 2 shows the selected study sites.

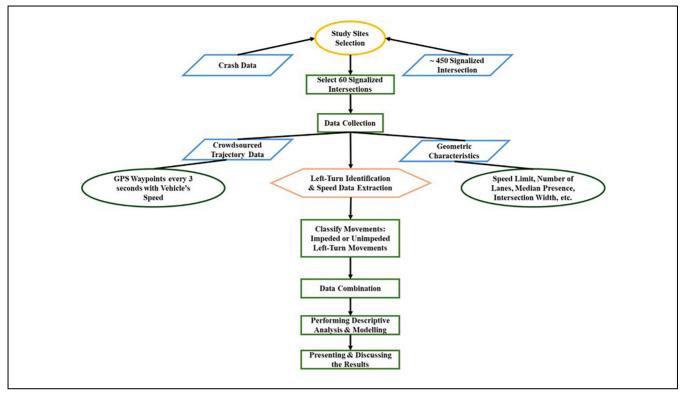


Figure 1. Study flow chart.

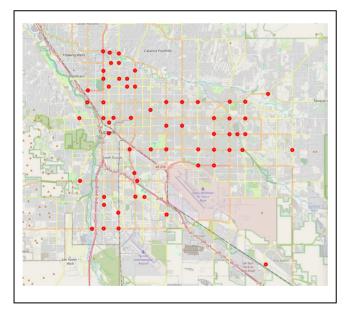


Figure 2. Study sites in Tucson, Arizona.

#### Data

Two data sets were used in this study including crowdsourced trajectory data and intersection geometric characteristics data. The crowdsourced trajectory data from WEJO were provided by the Pima Association of Governments (PAG). The crowdsourced trajectory data comprise individual vehicle waypoints reported every 3 s, with a positional accuracy within a 5-ft radius. Each waypoint includes attributes such as GPS location, timestamp, speed, heading, and an anonymous unique trajectory identifier. These crowdsourced trajectory data have been previously used by Khadka et al. (2022) to identify and analyze traffic congestion on both freeways and arterials (14). Similarly, Islam and Abdel-Aty (2023) employed crowdsourced trajectory data to predict real-time conflicts, using historical trajectory data of individual vehicles to assess potential future conflicts (15).

The geometric characteristics of the intersections were collected using Google Maps. The gathering of the geometric characteristics involved a manual extraction of these attributes, where a student used the measure tool to get information or street view to collect variables, such as the speed limit. These characteristics include the speed limit (SL), the number of left-turn lanes ( $N_l$ ), the presence of a median for that approach (Median), the distance required for a vehicle to traverse the intersection ( $W_L$ ), the width of the approaching lane ( $W_{Entry}$ ), and the width of the exit leg of the intersection ( $W_{Exit}$ ). In addition, the City of Tucson was consulted, and field visits were conducted to verify that the information extracted from Google Maps was accurate and accounted for any

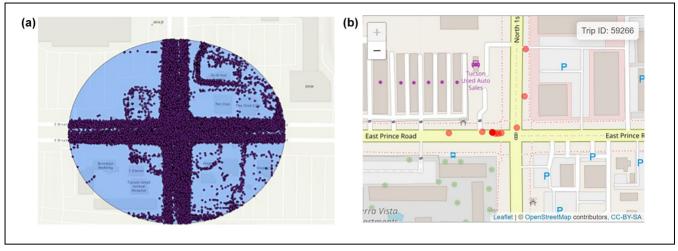


Figure 3. (a) All trajectory data for a specific intersection; and (b) example of identified eastbound left-turn movement.

updates or modifications not yet reflected in Google Maps.

# Left-Turning Vehicle Trajectories Extraction

This study evaluated factors influencing left-turning vehicle speeds. This section discusses the extraction of left-turn vehicle speed profiles. Figure 3a displays all trajectory data clipped for a specific intersection to identify left-turn movements. The identification process for left-turning movements involves three steps (16):

- Retrieving trajectory waypoints near the intersection to acquire entry and exit vehicle headings.
- Analyzing these headings to identify turning movement clusters and boundaries.
- 3. Filtering entire trajectories assigned to a particular movement based on the distance traveled.

Figure 3b illustrates an example of identified east-bound left-turn movement. The identification process was applied to the selected 60 intersections using data collected over 12 days in September and October 2021. In total, 32,884 left-turn trajectories were extracted from the data set.

In this study, left-turn movements were categorized as impeded or unimpeded movements. Previous studies analyzed primarily turning movements under free-flow conditions, with "free-flow" meaning: a vehicle having a minimum of 5-s leading headway and 3-s trailing headway (10), a passenger car's entire tangent section ahead, clear of other vehicles (17), and a vehicle without impacts with other vehicles, pedestrians, or cyclists (6). Impeded movements refer to left-turning maneuvers where the speed profile of the left-turning vehicle experiences delays because of conflicts with pedestrians, conflicting

approach traffic, or left-turn queues. Conversely, unimpeded movements are those speed profiles where the vehicle completes the left turn without stopping for queues or conflicts with pedestrians and crossing traffic. Figure 4a presents examples of unimpeded left-turning vehicles, while Figure 4b illustrates examples of impeded left-turning movements.

# Key Speed Profile Trajectory Points Identification

Speed values were gathered at three key points: at the entry of the intersection therefore named intersection entry speed ( $V_{\rm Entry}$ ); at the middle of the left-turning maneuver therefore named intersection mid-maneuver speed  $(V_M)$ ; and at the exit of the intersection therefore named intersection exit speed  $(V_{\text{Exit}})$ . Intersection entry speed  $(V_{\rm Entry})$  is defined as the speed of the vehicle at or very close to the stop bar. Intersection exit speed  $(V_{Exit})$  is defined as the speed of the vehicle as it exits the intersection and enters the adjacent leg. Mid-maneuver speed  $(V_M)$  is the speed of the vehicle between the entry and exit points. Additionally, for the vehicle's speed, two timestamps are taken, one before the entry speed is recorded and one after the exit speed is recorded. These are intersection pre-entry speed ( $V_{\text{Pre-Entry}}$ ) and intersection postexit speed ( $V_{\text{Post-Exit}}$ ), respectively.

#### **Descriptive Statistics**

This study collected key variables for vehicles making a left-turning maneuver: intersection entry speed, intersection mid-maneuver speed, and intersection exit speed. Figure 5 and Table 1 presents a summary of the statistics for the collected data, divided into two sections: one encompassing the entire data set, which includes both impeded and unimpeded movements; and the other



Figure 4. Examples of left-turning movements from trajectory points: (a) unimpeded left turn; and (b) impeded left turn.

focusing solely on unimpeded movements. Figure 5 illustrates the distribution of left-turning vehicle speeds for both left-turn movements and unimpeded-only movements, highlighting the median and 85th percentile speeds. For all left-turn movements, the pre-entry and entry speed distributions are right-skewed, reflecting variations and reduced speeds resulting from impeded movements. Specifically, for all left-turn movements, the median and 85th percentile speeds for pre-entry are

2.1 mph and 4.3 mph, and for entry speeds, they are 5.7 mph and 15 mph, respectively.

In contrast, the distribution of pre-entry and entry speeds for unimpeded movements differs, showing higher values. For unimpeded movements only, the median and 85th percentile speeds are significantly higher, with pre-entry speeds at 28.6 mph and 35.1 mph and entry speeds at 20.8 mph and 25.8 mph, respectively. The exit and post-exit speed distributions are

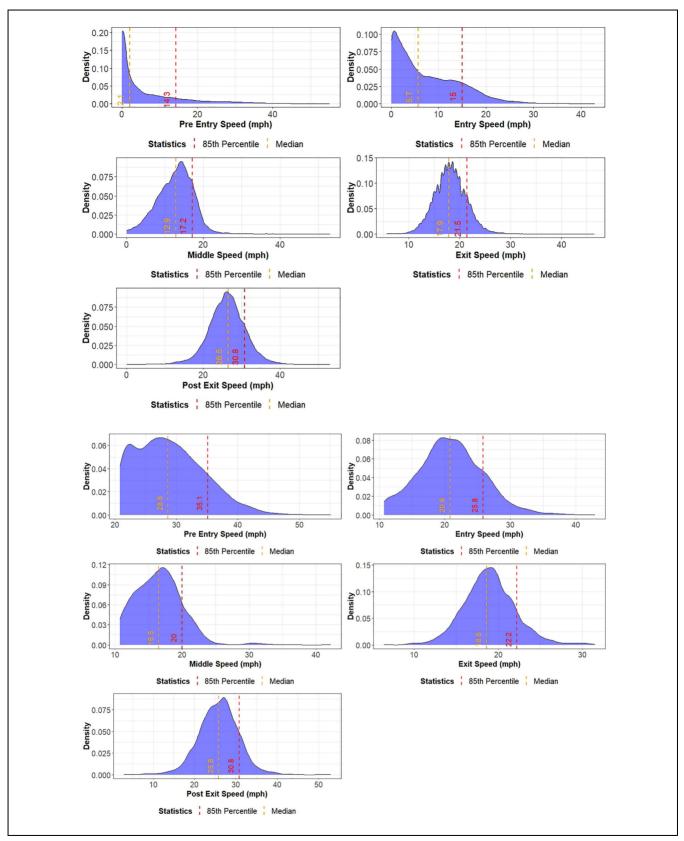


Figure 5. Distribution of and summary of left-turning vehicle speeds.

Table 1. Summary Statistics of left-turning vehicle speeds

Variable	No. of observations	Min.	Max.	Mean	Median
All (impeded +	unimpeded) left turns: 32,884 total o	bservations			
V <sub>Entry</sub>	32,884	0	42.9	7.35	5.7
V <sub>M</sub>	28,037	0	53	12.69	12.90
$V_{Exit}$	32,884	5.7	46.50	18.23	17.90
	turns: 1,979 total observations				
$V_{Entry}$	1,979	10.70	42.90	20.9	20.8
V <sub>M</sub>	1,178	10.7	42.2	16.56	16.50
$V_{Exit}$	1,979	4.40	31.50	18.97	18.60

Note: min. = minimum; max. = maximum.

Table 2. Summary of the Intersection's Geometric Attributes

Variable	Туре	Categories	Min.	Max.	Mean	Median	Frequency
Impeded + unimpeded movements: 32,884 observ	ations						
Speed limit (SL)	Integer	na	15	45	38.03	40	na
Number of left-turn lanes $(N_l)$	Categorical	1	na	na	na	na	18,842
na	na	2	na	na	na	na	14,042
Median presence	Categorical	Yes	na	na	na	na	22,535
na .	na	No	na	na	na	na	10,349
Distance required for a vehicle to traverse the intersection $(W_L)$	Continuous	na	73	256	143	141	na
Width of the approaching leg $(W_{Entry})$	Continuous	na	25	83	55.19	55	na
Width of the exit leg (W <sub>Exit</sub> )	Continuous	na	13	59	36.1	36	na
Unimpeded movements: 1,979 observations							
Speed limit (SL)	Integer	na	20	45	38.03	40	na
Number of left-turn lanes $(N_l)$	Categorical	I	na	na	na	na	1,176
Median presence	Categorical	2	na	na	na	na	803
na	na	Yes	na	na	na	na	1,251
Distance required for a vehicle to traverse the intersection $(W_L)$	Continuous	No	na	na	na	na	728
na	na	na	73	256	139.5	133	na
Width of the approaching lane $(W_{Entry})$	Continuous	na	25	83	55.01	55	na
Width of the exit leg of intersection $(W_{Exit})$	Continuous	na	13	59	34.53	32	na

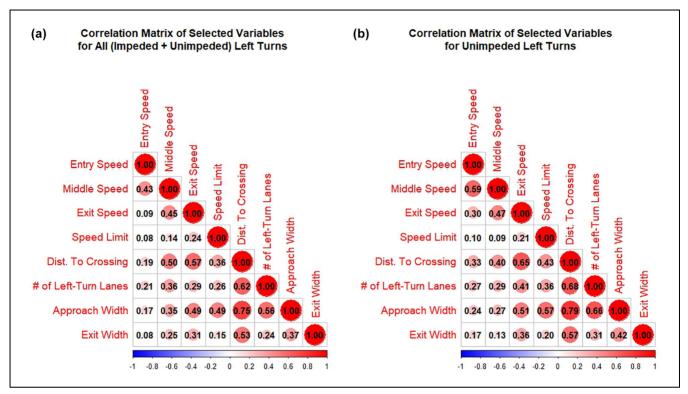
Note: min. = minimum; max. = maximum; na = not applicable.

similar for both all left-turn movements and unimpeded-only movements, but the median and 85th percentile values tend to be higher for unimpeded movements. Specifically, the median and 85th percentile for exit speeds are 17.9 mph and 21.5 mph for all left-turn movements, compared with 18.6 mph and 22.2 mph for unimpeded movements. For post-exit speeds, the values are 26.5 mph and 30.8 mph for all movements, while for unimpeded-only movements, they are 25.8 mph and 30.8 mph.

Yu et al. (2003) identified the top 10 parameters influencing yellow change and red clearance intervals for left-turn movements, based on a survey of researchers, practitioners, and executives. Among the top variables identified were the number of approaching and crossing lanes, visibility of traffic signals, speed limits, and average curve traversal speeds (18). In this study, variables such as intersection

length, width, and the number of lanes were prioritized because of their ease and reliability of measurement compared with more challenging factors such as intersection angle. Table 2 summarizes the geometric characteristics of intersections that were involved in the study according to the vehicle trajectories extracted from each intersection. It could be indicated that most of the intersections in the study had a median. It could also be observed that most of the trajectories were collected at intersections with one left-turn lane. Moreover, the average width of the approaching lanes was similar between the two data sets.

High correlations among input variables can adversely affect both model performance and the interpretability of results. To mitigate these issues, it is essential to eliminate highly correlated variables before proceeding with model development (19–21). In this study, the Pearson correlation test was applied to assess relationships between the



**Figure 6.** Correlation matrix of selected variables for: (a) all left-turn movements (impeded + unimpeded); and (b) unimpeded-only left-turn movements.

predictor variables and identify those with high collinearity (22). As illustrated in Figure 6, variables with correlation coefficients between 0.5 and 0.7 were considered moderately correlated, while those exceeding 0.7 were deemed strongly correlated (21, 23, 24). Therefore, the distance required for a vehicle to traverse the intersection ( $W_L$ ) was eliminated from model development.

# Methodology

Two models were used to identify geometric characteristics influencing left-turning vehicle speeds, including the OLS and quantile regression models. OLS regression models are commonly used to relate the mean of the response variable with its predictors. Unlike OLS regression, which estimates the mean, quantile regression estimates the relationships between correlates and specific percentiles, such as the 75th or 95th percentile. This approach allows for a more nuanced understanding of the factors influencing different points in the target variable distribution (25, 26). Quantile regression was used because the distributions of left-turning movements, particularly pre-entry and entry speeds, were right-skewed. Quantile regression has been observed to provide a more accurate prediction for skewed distributions of target

variables (27). The following section describes the OLS and quantile regression model in detail.

## **OLS Regression Model**

The OLS model is given by Equation 1:

$$y_i = \beta_0 + \sum_{i=1}^n \beta_j x_{ij} + \varepsilon_i \tag{1}$$

where

 $y_i$  = dependent variable, that is, speed of *i*th keypoint (mph), i = 1, 2, ..., m,

 $\beta_0$  = intercept,

 $\beta_j$  = coefficient of independent variable j, j = 1, 2, ..., n,  $x_{ij}$  = value of independent variables j in ith speed, and

 $\varepsilon_i$  = estimation error or residual for *i*th speed.

The error  $\varepsilon_i$  is assumed to be normally distributed with a mean of zero and a finite variance. Coefficients of the independent variables are estimated by minimizing the mean squared error criterion:

$$\sum_{i=1}^{m} \left( y_i - \beta_0 - \sum_{j=1}^{n} \beta_j x_{ij} \right)^2 \tag{2}$$

3.750

3.866

Movement Type	OLS	Median (50th percentile)	85th percentile
Entry speed			
Impeded + unimpeded	6.583	6.822	10.006
Unimpeded only	4.913	4.917	7.108
Middle speed			
Impeded + unimpeded	4.208	4.241	5.589
Unimpeded only	3.475	3.557	4.289
Exit speed			

2.622

2.759

Table 3. Model Prediction Measures (RMSE)

Note: OLS = ordinary least squares; RMSE = root mean squared error.

The resulting least squares estimates of  $\beta_0$  and  $\beta_0$  are denoted  $\hat{\beta}_0$  as and  $\hat{\beta}_j$ , respectively. OLS models provide intuitive estimations of the relationship between speed profiles and associated factors: a one-unit increase in an independent variable leads to an increase of  $\hat{\beta}_j$  in the mean speed, with all other variables held constant.

# Quantile Regression Model

Impeded + unimpeded

Unimpeded only

Unlike OLS regression, which models only the average speed profile, quantile regression can model the relationship of any quantile with a set of explanatory variables. In contrast to OLS models that minimize the mean squared error, quantile regression minimizes a sum that penalizes asymmetrically:  $(1-q)|\epsilon_i|$  for over predictions and  $q|\epsilon_i|$  for under predictions, where q represents the quantile point of the outcomes. For instance, to model the 85th percentile entry speed, q would be set to 0.85. The prediction errors in quantile regression are characterized by:

$$\sum_{i \ge \beta_0^q + \sum_{j=1}^n \beta_j^q x_{ij}}^n q \left| y_i - \beta_0^q - \sum_{j=1}^n \beta_j^q x_{ij} \right| + \sum_{i \le \beta_0^q + \sum_{i=1}^n \beta_j^q x_{ij}}^n (1-q) \left| y_i - \beta_0^q - \sum_{j=1}^n \beta_j^q x_{ij} \right|$$
(3)

where  $y_i$  is the dependent variable, representing the speed of the *i*th observation, and  $x_{ij}$  denotes the value of the *j*th independent variable in the *i*th observation.

#### Model Comparison

This study evaluated and compared the effectiveness of two modeling techniques (i.e., OLS and quantile regression) by assessing their ability to predict speeds using the root mean squared error (RMSE). A lower RMSE indicates more accurate predictions. The formula for RMSE is given by Equation 4:

$$RMSE = \sqrt{\frac{1}{n} \left( \sum_{i=1}^{n} y_i - \hat{y}_j \right)^2}$$
 (4)

where

n = number of observations,

2.622

2.797

 $y_i$  = observed speed for *i*th observation in the data set, and

 $\hat{y}_i$  = predicted speed for *i*th observation in the data set.

# **Results and Discussion**

OLS and quantile regression models were used to analyze the relationship between the speeds of left-turning vehicles and various influencing factors. The OLS model generates a single set of coefficients, reflecting the average change in speed with changes in the independent variables. In contrast, quantile regression offers distinct sets of coefficients for each quantile, providing a more nuanced understanding of how independent variables influence speeds across different points in the speed distribution. For each quantile, the coefficient interpretation mirrors that of the OLS model, representing the change in speed for that specific quantile category. The following sections provide detailed results of the entry speed, middle-of-maneuver speed, and exit speed analyses.

#### Model Assessment

The performance of the models was assessed using RMSE. The lower the RMSE value, the better the model's predictive accuracy. Table 3 shows the results of the model assessment. For entry speed, the OLS model had the lowest RMSE (6.583) among models that were fit when considering all left-turn (impeded and unimpeded) movements. Similarly, for unimpeded left-turn movements only, the OLS model had the lowest RMSE (4.913), indicating better predictive performance in the unimpeded context. For middle-of-maneuver speed, the OLS achieved the lowest RMSE (4.208 and 3.475) for

Table 4. Estimation results of OLS and Quantile Regression Models for Entry Speed

	OLS		Median (50th percentile)		85th percentile		
Variables	Coefficient	P-Value	Coefficient	P-Value	Coefficient	P-Value	
All left turns							
Intercept	5.276	0.00	2.647	0.00	12.698	0.00	
Speed limit	0.024*	0.03	0.038*	0.01	0.008	0.67	
$\dot{N_L}$							
٦	Base	e	Base	e	Base	2	
2	1.330*	0.00	2.187*	0.00	1.783*	0.00	
Median							
No	Base	9	Base	е	Base	2	
Yes	-0.869*	0.00	-0.223	0.22	-2.291*	0.00	
$W_{Entry}$	0.036*	0.00	0.017*	0.03	0.088*	0.00	
W <sub>Exit</sub>	-0.02*	0.00	-0.004	0.51	-0.062*	0.00	
RMSE	6.541		6.782		9.935		
Unimpeded left turi	ns						
Intercept	18.558	0.00	17.967	0.00	20.780	0.00	
Speed limit	0.001	0.97	-0.001	0.98	0.017	0.78	
$\dot{N_L}$							
_ I	Base	9	Base	е	Base	2	
2	1.319*	0.00	1.558*	0.00	1.894*	0.00	
Median							
No	Base	9	Base	е	Base	2	
Yes	-0.837*	0.03	-0.370	0.42	-0.171	0.82	
$W_{Entry}$	0.017	0.25	0.014	0.45	0.030	0.28	
W <sub>Exit</sub>	0.037*	0.02	0.048*	0.01	0.044	0.11	
RMSE	4.75		4.77			6.561	

Note: OLS = ordinary least squares; RMSE = root mean squared error.

the model fit to all left-turn (impeded and unimpeded) movements and unimpeded left-turn movements only. For exit speed, the OLS model had the lowest RMSE of 2.622 and 2.759 for the combined data set and the unimpeded movements. Overall, the RMSE values indicate that the OLS and quantile regression models provide reasonably accurate predictions, with the quantile regression models offering nuanced insights into different percentiles of speed distributions. The models performed better for unimpeded movements, reflecting the less variable conditions in such scenarios.

# Entry Speed

All Left-Turn Movements. Table 4 presents the results of the OLS and quantile regression models estimated at the 50th and 85th percentiles for intersection entry speed. All the selected variables except the presence of median at the 50th percentile were found to significantly influence entry speed. For all left-turn movements (i.e., impeded and unimpeded) the speed limit shows a statistically significant positive correlation with entry speed in the OLS and 50th percentile models but not at the 85th percentile. This suggests that the influence of posted speed limits is

more apparent under average and typical conditions. The number of left-turn lanes was found to be a critical factor influencing entry speed. The presence of two leftturn lanes is consistently and significantly associated with higher entry speeds across all models. These results reinforce the need to consider lane configuration in the design of yellow intervals for left-turning movements. The presence of a median has a significant negative association with entry speed in the OLS and 85th percentile models, indicating that medians can reduce higher-speed entries into intersections. Approach width  $(W_{\text{Entry}})$  is positively and significantly associated with entry speed across all three models, indicating that wider intersection entries may encourage higher vehicle entry speeds. In contrast, exit width  $(W_{Exit})$  is significantly and negatively correlated with entry speed in the OLS and 85th percentile models, suggesting that wider downstream paths reduce intersection entry speed.

Unimpeded Left-Turn Movements. For unimpeded movements, the speed limit does not show a statistically significant relationship with entry speed in any of the models. This indicates that, in the absence of vehicle interference, driver entry speeds are likely to be influenced more by

<sup>\*</sup>Statistically significant at 0.05 level.

Table 5. Estimation Results of OLS and Quantile Regression Models for Middle-of-Maneuver Sp
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	OLS		Median (50th percentile)		85th percentile	
Variables	Coefficient	P-Value	Coefficient	P-Value	Coefficient	P-Value
All left turns						
Intercept	7.494	0.00	5.626	0.00	11.508	0.00
Speed limit	-0.032*	0.00	-0.022*	0.00	-0.024*	0.00
$\dot{N_L}$						
١	Base		Base	<b>.</b>	Base	<b>:</b>
2	2.338*	0.00	2.487*	0.00	1.847*	0.00
Median						
No	Base		Base	<b>.</b>	Base	
Yes	0.846*	0.00	0.928*	0.00	0.462*	0.00
$W_{Entry}$	0.044*	0.00	0.058*	0.00	0.054*	0.00
W <sub>Exit</sub>	0.066*	0.00	0.088*	0.00	0.048*	0.00
RMSE	4.335		4.364		5.741	
Unimpeded left turn	ns					
Intercept	16.451	0.00	16.457	0.00	16.759	0.00
Speed limit	-0.089*	0.01	-0.140*	0.00	-0.038	0.36
$N_L$						
١	Base		Base		Base	
2	1.436*	0.00	1.570*	0.00	1.485*	0.01
Median						
No	Base		Base	<b>.</b>	Base	<b>:</b>
Yes	-0.306	0.46	-0.510	0.25	-0.440	0.41
$W_{Entry}$	0.051*	0.00	0.070*	0.00	0.058*	0.01
W <sub>Exit</sub>	0.001	0.98	0.026	0.11	0.005	0.79
RMSE	3.515		3.53		4.515	

Note: OLS = ordinary least squares; RMSE = root mean squared error.

roadway geometry or individual driving behavior than posted limits. The number of left-turn lanes remains a significant factor, with dual-lane configurations associated with higher entry speeds, 1.32 mph (OLS), 1.56 mph (50th percentile), and 1.89 mph (85th percentile). These consistent findings further highlight the role of lane configuration even under unimpeded conditions. The presence of a median is statistically significant in the OLS model, showing a negative correlation with entry speed. This suggests that medians may influence entry behavior when there is no vehicle interaction, although the effect appears insignificant across percentiles. In contrast to all left-turn movements, approach width ( $W_{\rm Entry}$ ) is not statistically significantly related to entry speed under unimpeded conditions. However, exit width  $(W_{Exit})$  is significantly and positively associated with entry speed in both the OLS and 50th percentile models, implying that wider exit paths may encourage faster entries when the turning movement is not impeded.

# Middle-of-Maneuver Speed

All Left-Turn Movements. Table 5 presents the OLS and quantile regression models for the middle-of-maneuver speed. The speed limit shows a consistently significant

negative association with middle-of-maneuver speed across all three models. This suggests that at intersections along higher-speed arterials, drivers tend to reduce their speed mid-turn, possibly as a result of greater caution when finding gaps in faster-moving cross traffic. The number of left-turn lanes remains a significant factor in all models. When two left-turn lanes are present, drivers tend to maintain higher speeds during the maneuver. This likely reflects greater geometric flexibility and confidence in lane positioning during the turn. The presence of a median shows a significant positive relationship with middle-of-maneuver speed across all models. This may indicate that medians help organize traffic flow and reduce uncertainty, enabling smoother and faster midturn movement. Approach width  $(W_{\text{Entry}})$  and exit width  $(W_{\rm Exit})$  have positive and statistically significant associations with middle-of-maneuver speed across all models, differing from prior results. Wider entry and exit paths may reduce turning radii or allow more fluid lane changes mid-turn, contributing to increased speeds during the maneuver.

*Unimpeded Left-Turn Movements.* For unimpeded left turns, the speed limit retains a negative correlation with the

<sup>\*</sup>Statistically significant at 0.05 level.

Table 6. Estimation Results of OLS and Quantile Regression Models for Exit Speed

	OLS		Median (50th percentile)		85th percentile	
Variables	Coefficient	P-Value	Coefficient	P-Value	Coefficient	<i>P</i> -Value
All left turns						
Intercept	11.850	0.00	11.581	0.00	14.343	0.00
Speed limit $N_L$	0.007	0.12	0.012*	0.01	0.006	0.22
Ī	Base	2	Base	9	Base	e
2	-0.050	0.28	-0.155*	0.00	-0.026	0.66
Median						
No	Base		Base		Base	
Yes	1.086*	0.00	0.997*	0.00	1.159*	0.00
$W_{Entry}$	0.067*	0.00	0.072*	0.00	0.073*	0.00
W <sub>Exit</sub>	0.045*	0.00	0.041*	0.00	0.041*	0.00
RMSE	2.792		2.794		3.883	
Unimpeded left turns	S					
Intercept	14.068	0.00	13.599	0.00	15.748	0.00
Speed limit	-0.013	0.52	0.009	0.64	-0.031*	0.02
$\dot{N_L}$						
1	Base	9	Base		Base	
2	-0.156	0.47	-0.442*	0.04	-0.828*	0.00
Median						
No	Base	9	Base	e	Base	e
Yes	0.555	0.02	0.126	0.58	1.054*	0.00
$W_{Entry}$	0.062*	0.00	0.069*	0.00	0.077*	0.00
$W_{Exit}$	0.051*	0.00	0.040*	0.00	0.071*	0.00
RMSE	2.84	4	2.857		3.850	

Note: OLS = ordinary least squares; RMSE = root mean squared error.

middle-of-maneuver speed, significant in the OLS and 50th percentile models but not at the 85th percentile. This again reinforces the idea that drivers tend to slow more on high-speed arterials when navigating unimpeded turns, potentially exercising caution even without traffic interference. The number of left-turn lanes continues to have a positive and significant effect on speed across all models. This suggests that even under free-flowing conditions, dual-lane turns facilitate more confident, faster maneuvering. In contrast to the all left-turn movements, the presence of a median is not statistically significant in any model for unimpeded movements. The approach width  $(W_{\rm Entrv})$  remains positively and significantly associated with speed in all models, reinforcing the idea that wider entry geometry supports more fluid turning. Meanwhile, exit width  $(W_{Exit})$  is not statistically significantly related to middle-of-maneuver speed in any model, indicating it may not influence driver behavior as strongly when turns are made without vehicle conflict.

# Exit Speed

All Left-Turn Movements. Table 6 summarizes the results of the OLS and quantile regression models for exit speed. For all left-turn movements, the speed limit shows a small positive relationship with exit speed, but it is statistically significant only in the 50th percentile model. This suggests that drivers tend to exit slightly faster on higherspeed roadways, particularly around the median speed distribution. The number of left-turn lanes is negatively associated with exit speed in all models, though it is statistically significant only in the 50th percentile model. This supports the idea that in dual-lane configurations, drivers may slow down while exiting because of constraints in lane positioning or the presence of adjacent vehicles. The presence of a median is positively and significantly associated with exit speed across all models. This may indicate that medians reduce perceived or actual conflict points at intersections, allowing drivers to accelerate more confidently when completing their turn. Both approach width  $(W_{\text{Entry}})$  and exit width  $(W_{\text{Exit}})$  are positively and significantly correlated with exit speed in all models. This differs from earlier results where  $W_{\rm Exit}$ showed a negative correlation. The updated models suggest that wider entry and exit lanes offer more maneuvering space, encouraging higher exit speeds.

Unimpeded Left-Turn Movements. Among unimpeded left turns, the speed limit is not significantly associated with

<sup>\*</sup>Statistically significant at 0.05 level.

exit speed in the OLS or 50th percentile models but shows a significant negative effect at the 85th percentile. This may indicate that faster drivers self-regulate more conservatively when approaching exits on high-speed roads in the absence of traffic interference. The number of left-turn lanes continues to have a negative and significant effect on exit speeds across higher quantiles, with a strong negative association at the 85th percentile. This further supports the hypothesis that dual left-turn lanes constrain higher-speed exits, especially among faster drivers. Unlike all left-turn movements, the presence of a median is only significant in the 85th percentile model. This suggests that median presence primarily benefits higher-speed turn executions in unimpeded conditions, likely by creating a clearer path or visual cue for acceleration. Both  $W_{\text{Entry}}$  and  $W_{\text{Exit}}$  remain significantly and positively correlated with exit speed across all models, reinforcing their roles in providing geometric comfort and flexibility that support faster turning completion.

#### **Conclusions**

Determining yellow and red clearance intervals requires parameters such as perception-reaction time, the 85th percentile of approach speed, and approach grade. While some can be applied universally, local data are needed for specific parameters such as left-turn entry speed. The ITE 2020 Guidelines and NCHRP Report 731 suggest using speed limits to estimate the 85th percentile of intersection approach speed and intersection entry speed, but this method may lead to inaccuracies at intersections with unique geometric features, such as dual left-turn lanes or varying turning radii. While collecting field speed data provides accurate measurements, it poses challenges for many agencies because of cost and scalability. This study aims to evaluate how geometric characteristics at signalized intersections affect left-turning vehicle speeds and demonstrates the usefulness of crowdsourced trajectory data in determining left-turn entry speed. The research also considers speeds during the middle and exit phases of the left-turn maneuver, offering valuable data for various applications such as driving simulators and traffic management tools.

In this study, 60 signalized intersections within the City of Tucson were selected to examine left-turning vehicle speeds. Using crowdsourced trajectory data, left-turning vehicles were identified, and their speeds recorded. Unlike previous studies, which focused primarily on free-flow conditions, left-turn movements were categorized into two types: impeded and unimpeded. Impeded movements include left-turning maneuvers where the speed profile is impeded as a result of delays caused by conflicts with pedestrians, conflicting approach traffic, or left-turn queues. Unimpeded movements, on

the other hand, refer to left turns with unimpeded speed profiles, where the vehicle completes the turn without stopping for queues or conflicts with pedestrians and crossing traffic. OLS regression models and quantile regression models were used to analyze the speed profiles and determine the impact of certain variables on left-turning speeds. These models provided insights into how different factors influence left-turning speeds under both impeded and unimpeded conditions.

The results indicate that roadway geometry and regulatory factors significantly influence vehicle speeds during left-turn maneuvers. An increased number of left-turn lanes was associated with higher entry and mid-maneuver speeds in impeded movements but corresponded with lower exit speeds across both impeded and unimpeded conditions. The presence of a median reduced entry speeds but was linked to increased mid-maneuver and exit speeds in impeded scenarios. Additionally, wider approach widths were positively associated with entry and exit speeds for impeded movements, while exit width was a significant factor in increasing exit speeds regardless of movement type.

These findings highlight the importance of considering geometric and operational characteristics of intersections to improve signal timing design for left-turning vehicles at intersections. Based on these results, it is imperative that agencies consider collecting data in the field to estimate the vellow interval, especially at intersections with unique characteristics. As it was indicated that number of lanes and presence of median have a major impact on the entry speed. This result also indicates that agencies could use the crowdsourced trajectory data to estimate the vehicle entry speeds and apply them to estimate the yellow intervals because these data capture the variability of the left-turn entry speeds at different locations. Although the most common intersections in Tucson are standard 90° intersections, a limitation of this study is that it does not consider intersection angle as a variable. Moreover, it is important to note the limitations of crowdsourced data, as they do not represent the entire population of road users, and penetration rates vary between agencies. Although yellow and red clearance intervals are fixed in the City of Tucson, some jurisdictions use variable signal timing strategies, including dynamic yellow change and clearance intervals. Future research should, therefore, explore the effectiveness and feasibility of such adaptive systems under varying traffic conditions. Future research directions could include examining the impact of other vehicles, such as following or leading vehicles, and incorporating signal timing phases, including permissive or protected left-turn phases, into the analysis. Additionally, accounting for the influence of intersection angle, pedestrians, cyclists, and heavy vehicles, along with demographic

characteristics, vehicle classification, and other relevant factors could provide a more comprehensive understanding of driver behavior for left-turn movements. By expanding the scope of the study, researchers can better inform intersection design and traffic management strategies to enhance overall traffic safety and performance.

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# **Authors' Contribution**

The authors confirm contribution to the paper as follows: study conception and design: Pouya Jalali Khalilabadi, Henrick Haule, data collection: Pouya Jalali Khalilabadi, Henrick Haule; analysis and interpretation of results: Pouya Jalali Khalilabadi, Henrick Haule; draft manuscript preparation: Pouya Jalali Khalilabadi, Henrick Haule, Yao-Jan Wu. All authors reviewed the results and approved the final version of the manuscript.

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